

# Computational and experimental approaches in neuronal morphogenesis and network formation

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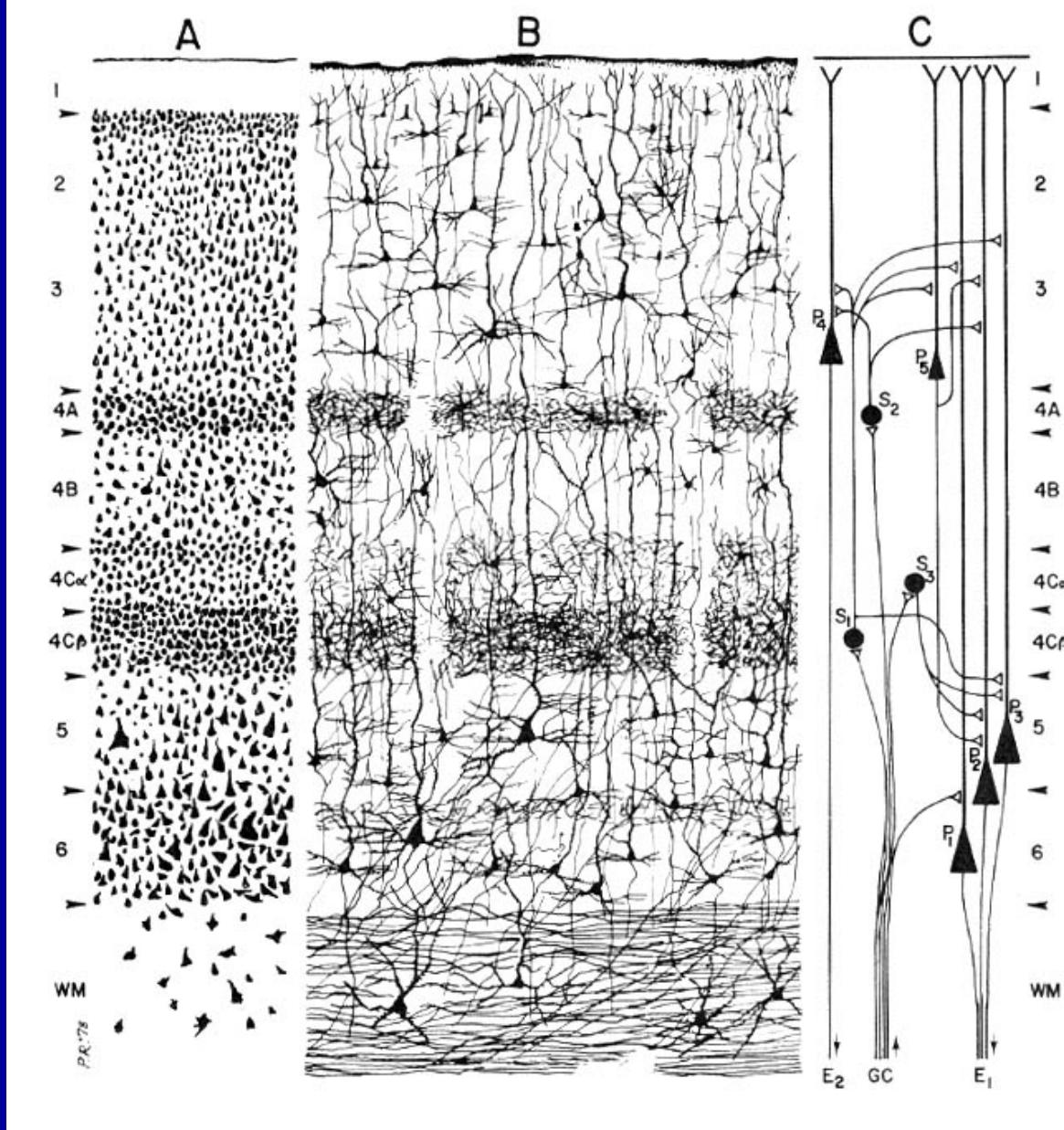
- Introduction
- Modeling dendritic morphological complexity
- Development of synaptic connectivity
- Spontaneous bioelectric activity in developing neuronal networks *in vitro*
- Balance of excitation and inhibition

# Nervous system

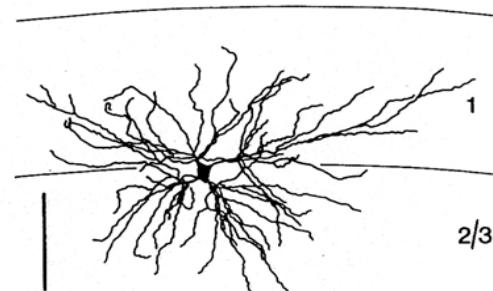
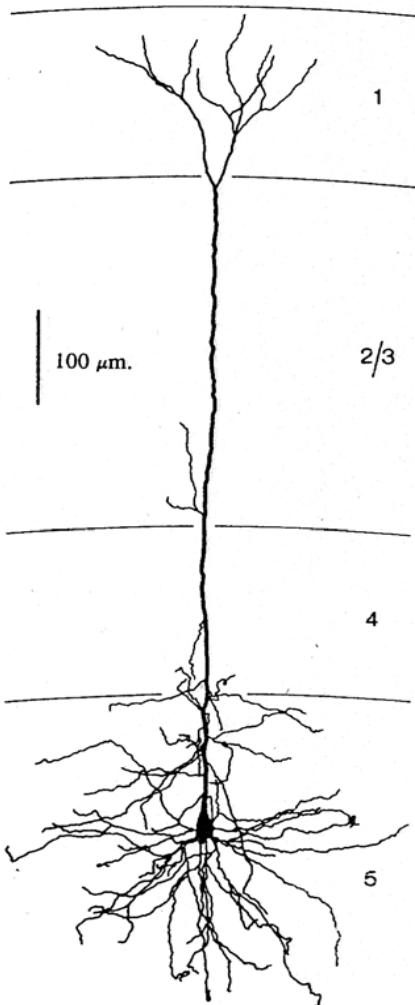
- neurons
- synaptic connections
- neuronal networks

## Cortical organization

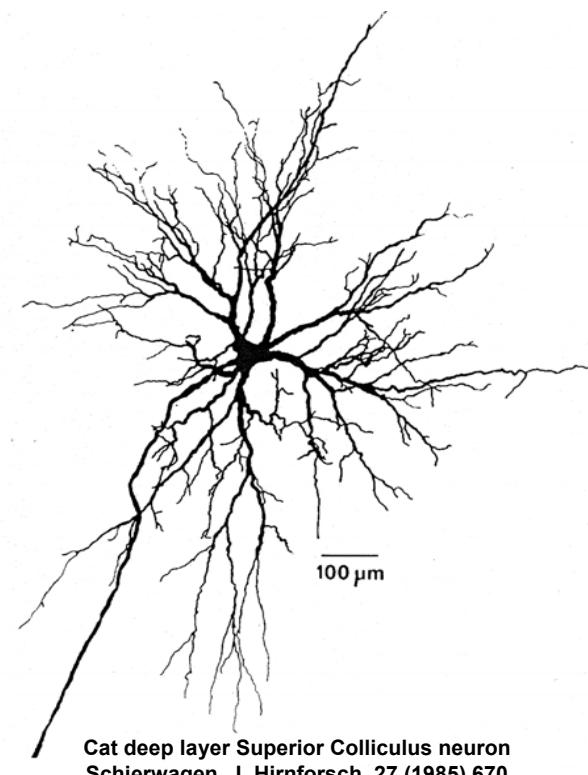
- Cell bodies
- Dendrites and axons
- Synaptic connections
- Layers
- Excitation - Inhibition



# Neuronal morphological diversity



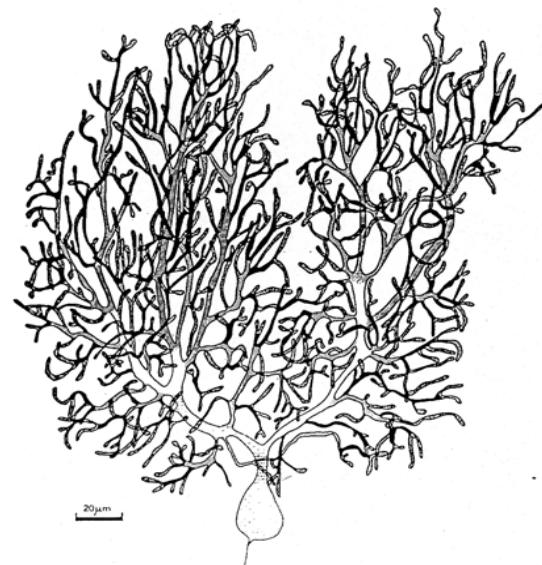
Rat visual cortex layer 2/3 pyramidal neuron  
Larkman and Mason. J. Neurosc. 10 (1990) 1407



Cat deep layer Superior Colliculus neuron  
Schierwagen, J. Hirnforsch. 27 (1985) 670



Cat hindlimb Moto neuron - Uhlfhake and Cullheim  
J. Comp. Neurol. 278 (1988) 88

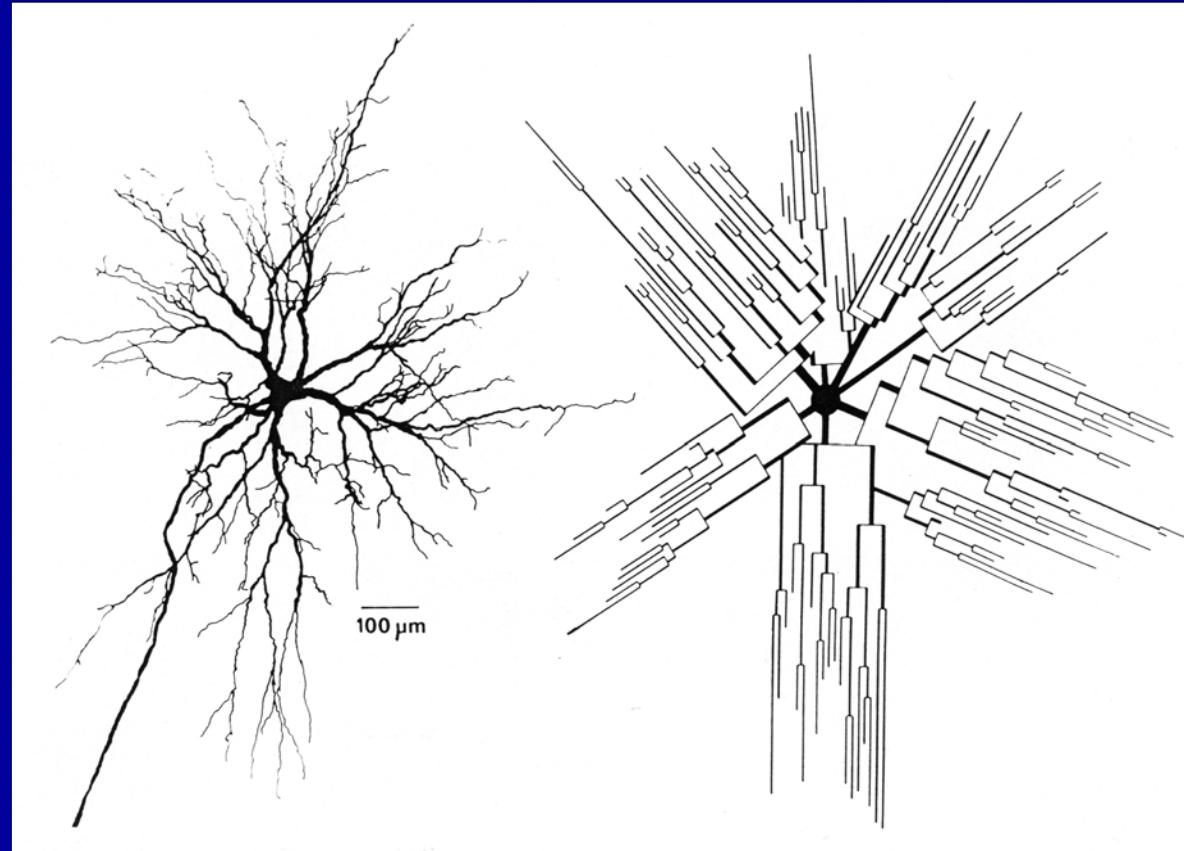


Rat visual cortex layer 5 pyramidal neuron  
Larkman and Mason, J. Neurosc. 10 (1990) 1407

Rat cerebellar Purkinje cell - Berry and Bradley  
Brain Res. 112 (1976) 1-35.

# Shape parameters of dendritic branching patterns

- Number of segments
- Length of segments
  - Diameter of segments
- Topological structure
  - Curvature of segments
  - 3D embedding



Cat deep layer superior colliculus neuron  
A. Schierwagen J. Hirnf. 27 (1986) 679

# Modeling dendritic morphological complexity using principles of neuronal development

Quantification of dendritic morphological variability

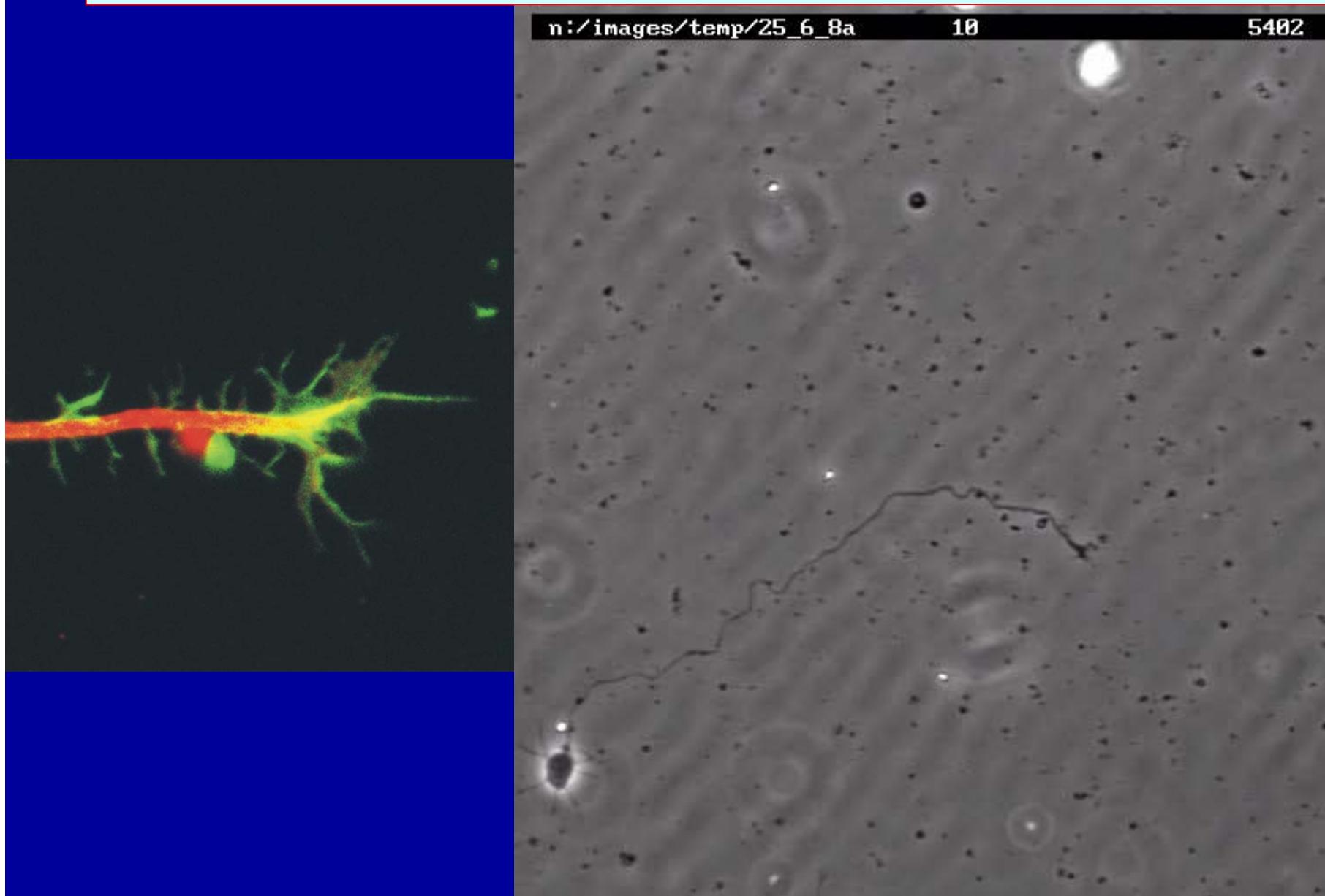
Source of morphological variability

Morphological differences between cell types

‘Design rules’ of dendritic morphology

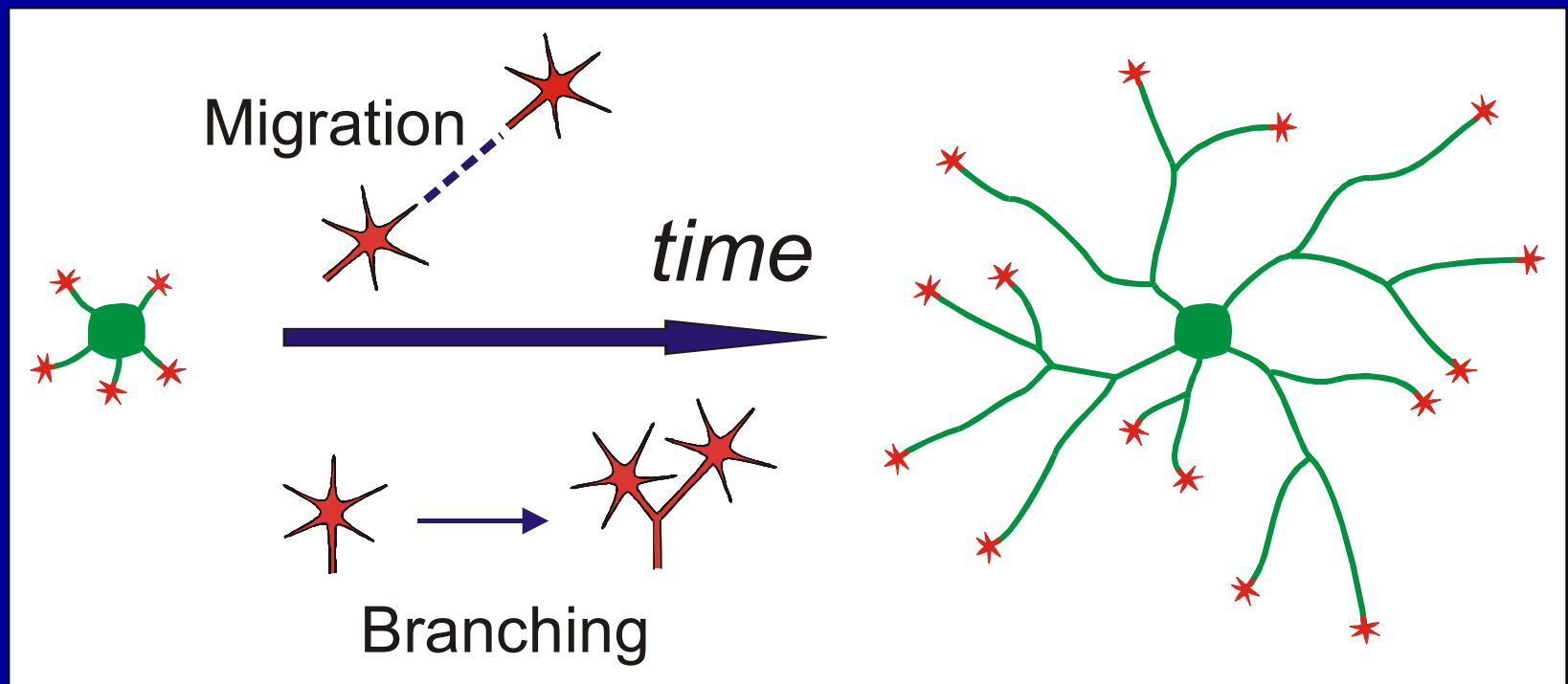
Functional implications of dendritic shape characteristics

# Axonal and dendritic outgrowth through growth cone elongation, branching, and retraction

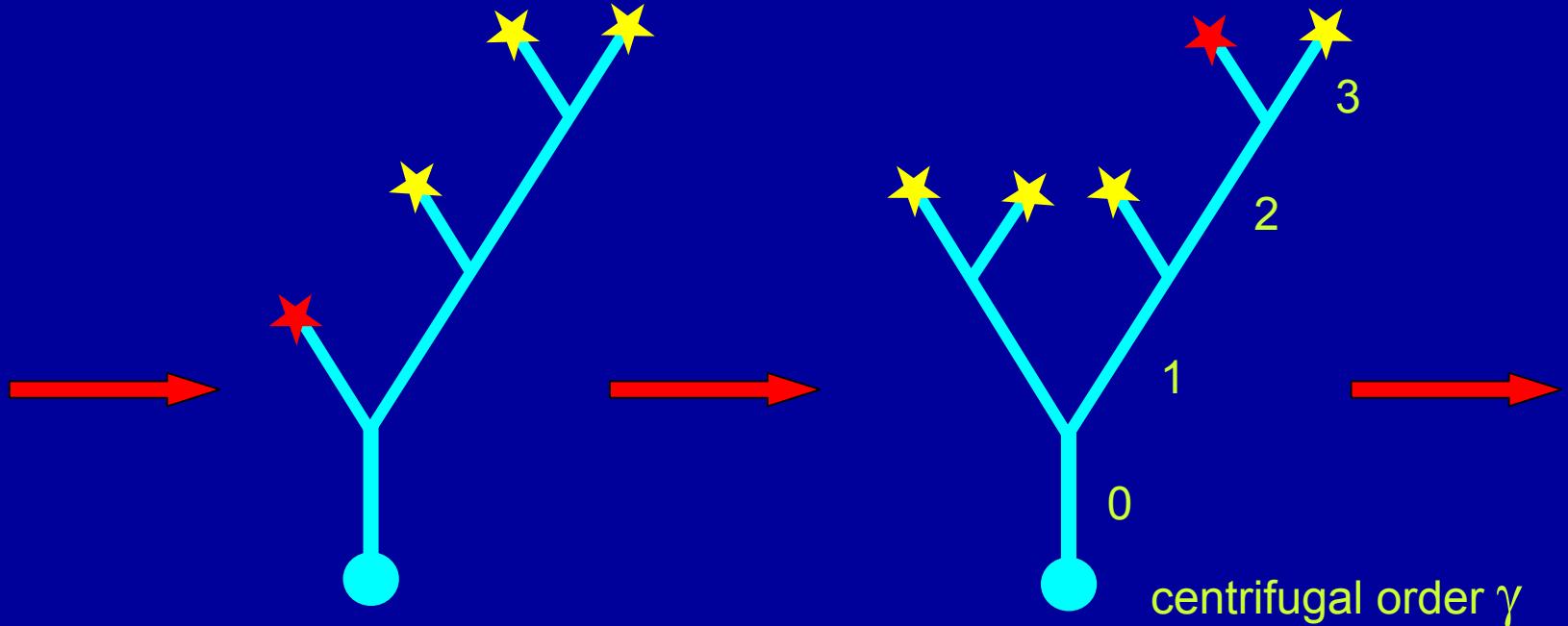


# Modeling dendritic morphological complexity

*find minimal phenomenological rules for reproducing dendritic morphological complexity*



## Random selection of segment for branching

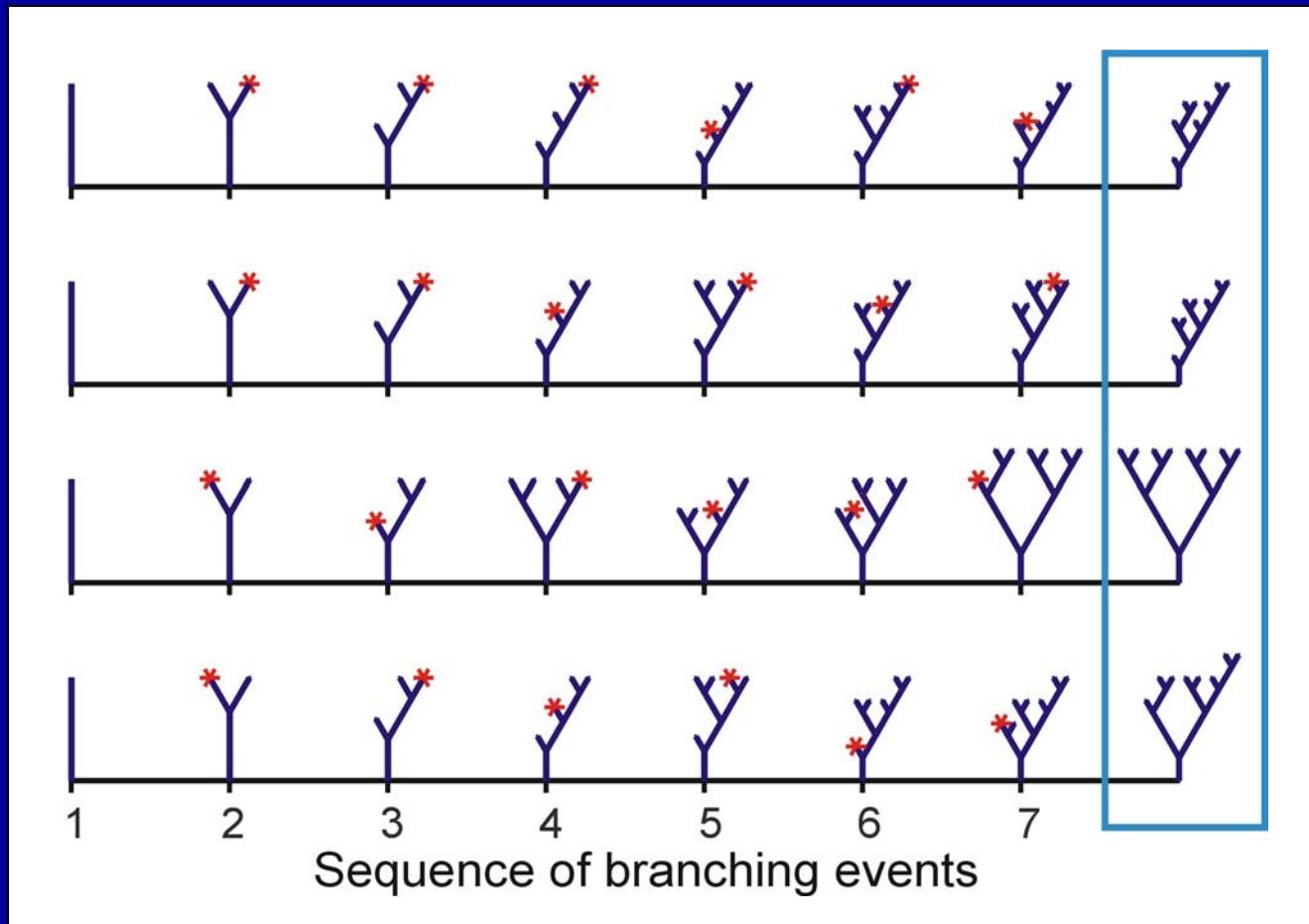


$$p_i \approx 2^{-S\gamma_i} / C$$

Selection probability of a terminal segment for branching depends on its centrifugal order

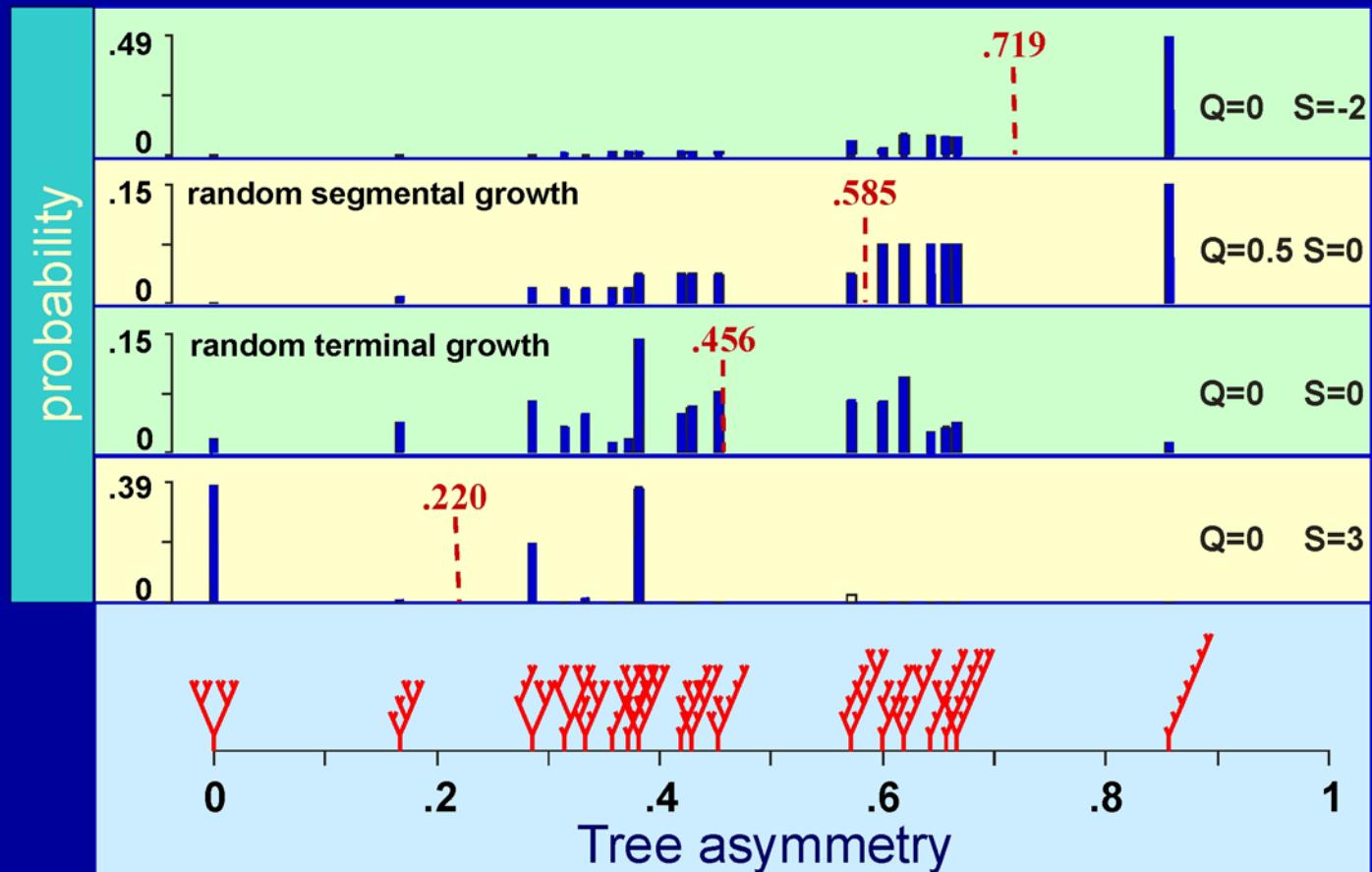
# Topological variation due to random branching

$S=0$     $p_i = 1/n$    equal selection probabilities

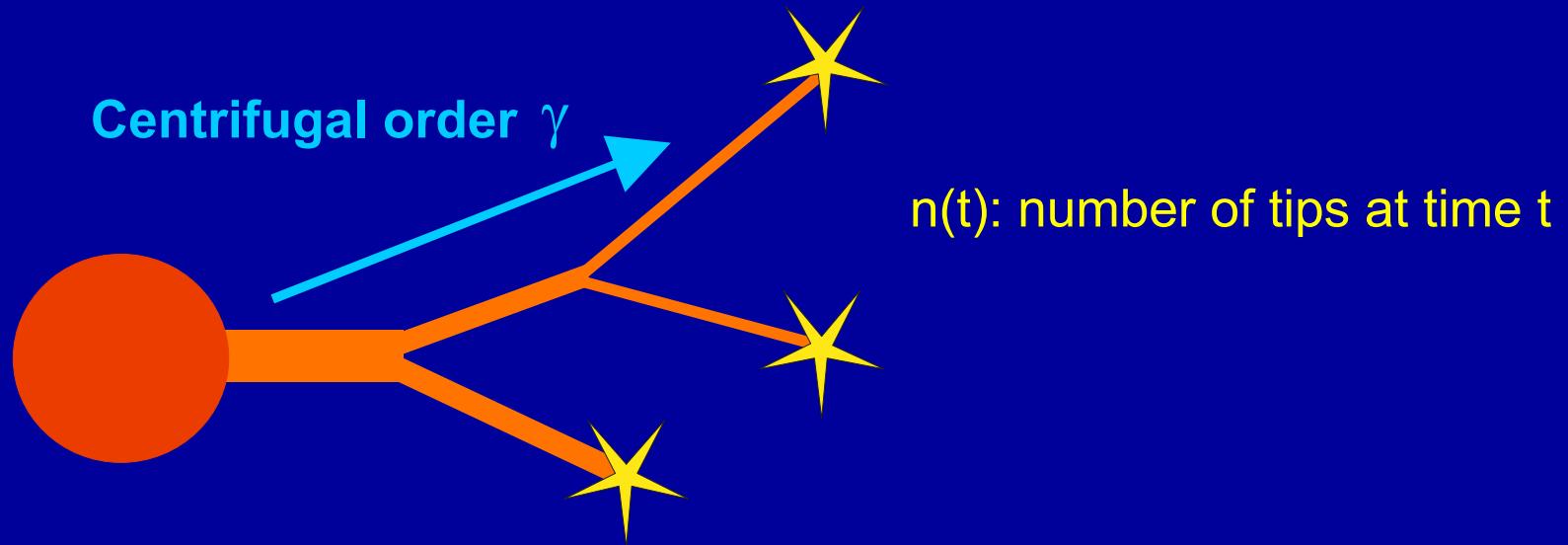


# Tree-type probabilities depend on mode of branching

Trees with 8 terminal segments



# Branching in continuous time



Branching probability per unit  
of time of a tip:

E: “competition parameter”

S: modulation of order dependency

C: normalization factor

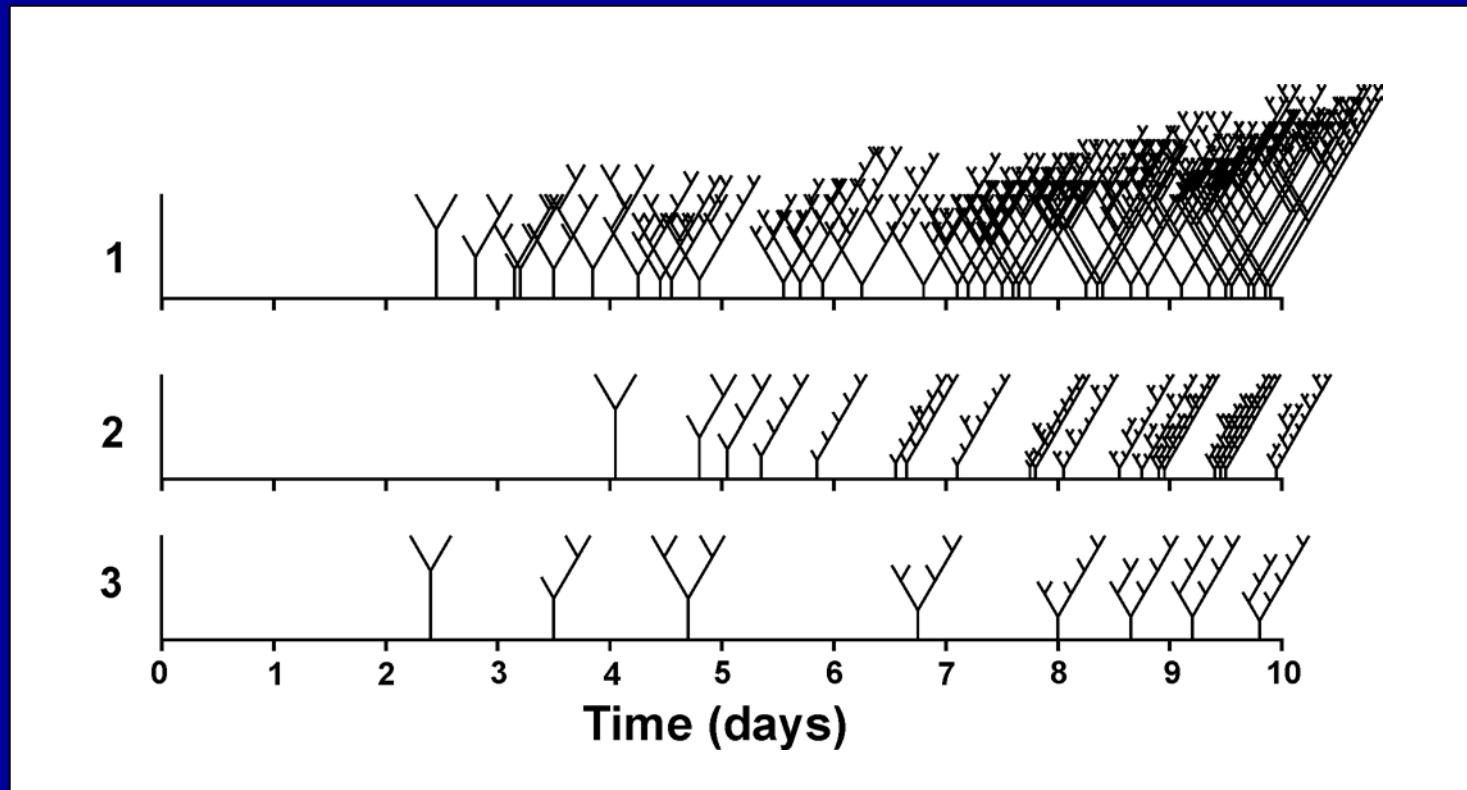
$$p_b(t) = D(t) \cdot n(t)^{-E} \cdot \left\{ 2^{-S\gamma} / C \right\}$$

D(t): baseline branching rate

$$B(t) = \int_0^t D(s) ds$$

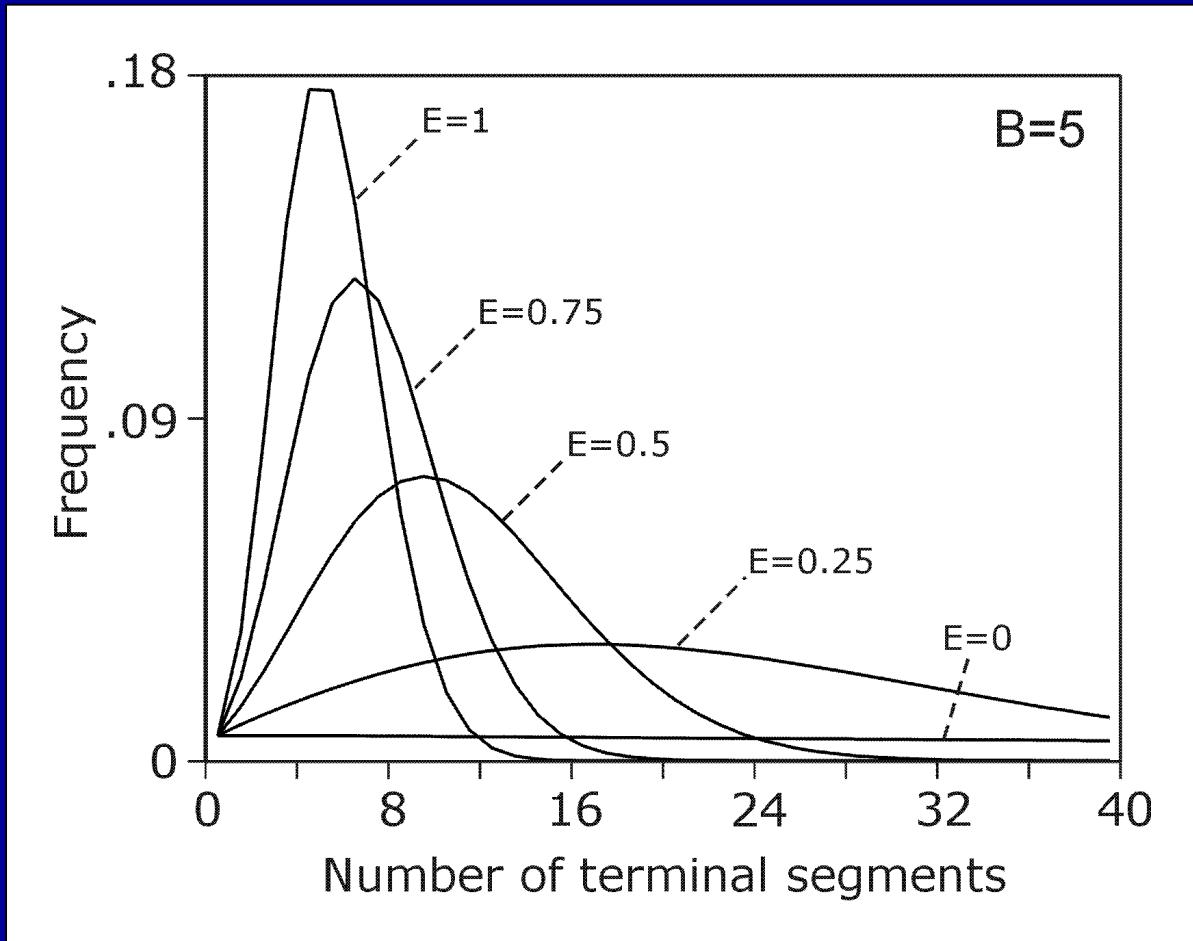
## Random branching sequences

$E = 0$  (no competition) ♦  
 $B = 3$     $D = B / 10 = 0.3 / \text{day}$  (constant)



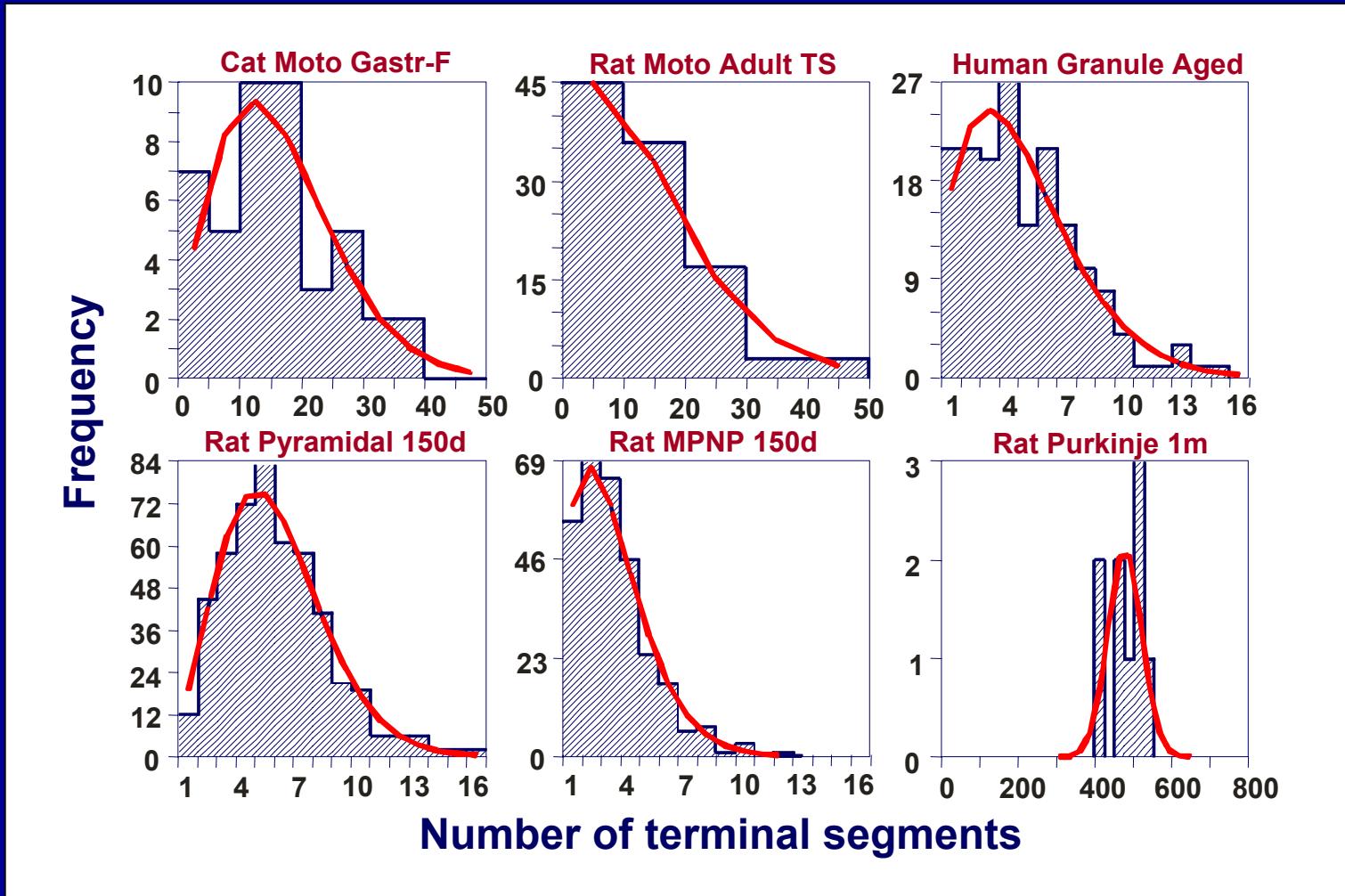
♦ *Unrestricted proliferation of the number of terminal segments*

## Competition parameter E determines shape of terminal segment number distribution

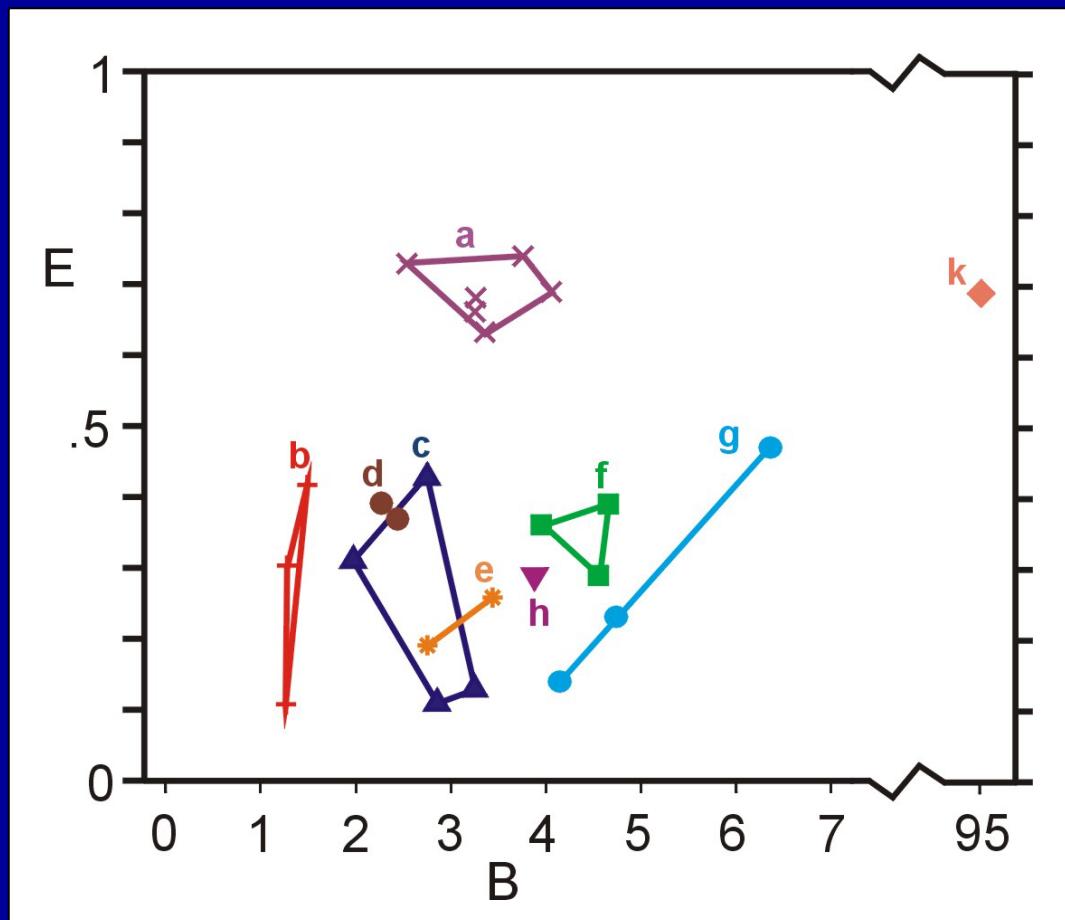


# Variation in terminal segment number

Matching between distributions of observed and model trees



## Optimized branching parameters B and E



- a - rat cortical pyramidal neurons
- b - rat cortical multipolar nonpyramidal
- c - rat motoneurons
- d - human dentate granule
- e - cultured cholinergic interneurons
- f - cat motoneurons
- g - frog motoneurons
- h - cat deep layer superior colliculus
- k - guinea pig cerebellar Purkinje cells

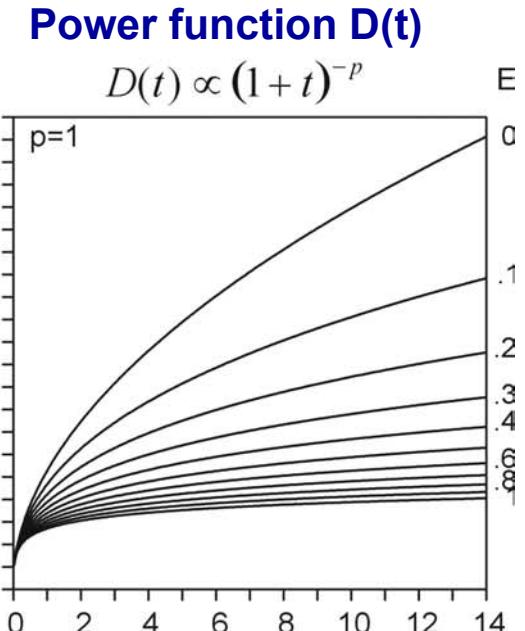
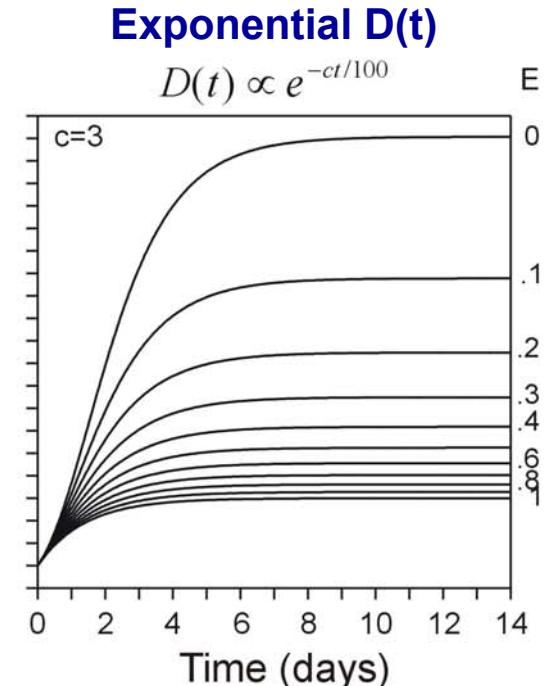
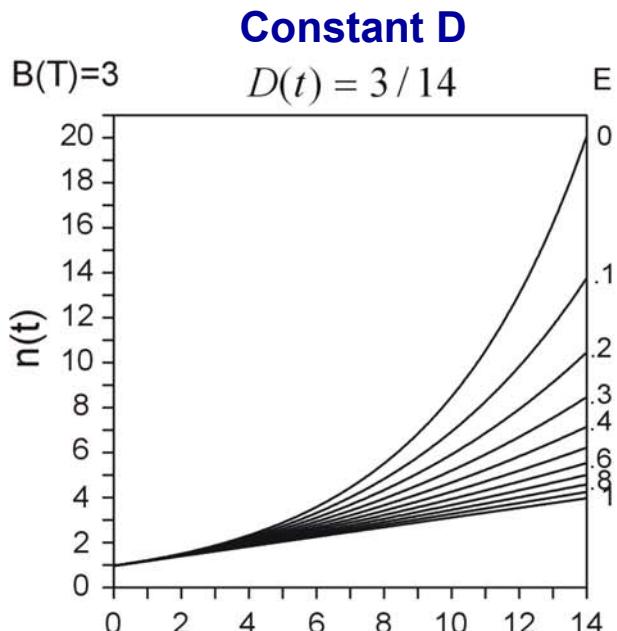
## Growth function $n(t)$ - influence of baseline branching rate function $D(t)$

$$\frac{dn(t)}{dt} = D(t)n(t)^{1-E}$$

$$n(t) = [1 + EB(t)]^{1/E}$$

$$n(t \mid E = 0) = e^{B(t)}$$

$$B(t) = \int_0^t D(s)ds$$



## Growth curve for the number of tips $n(t)$

Rat multipolar nonpyramidal dendritic trees  
(Parnavelas, Uylings)

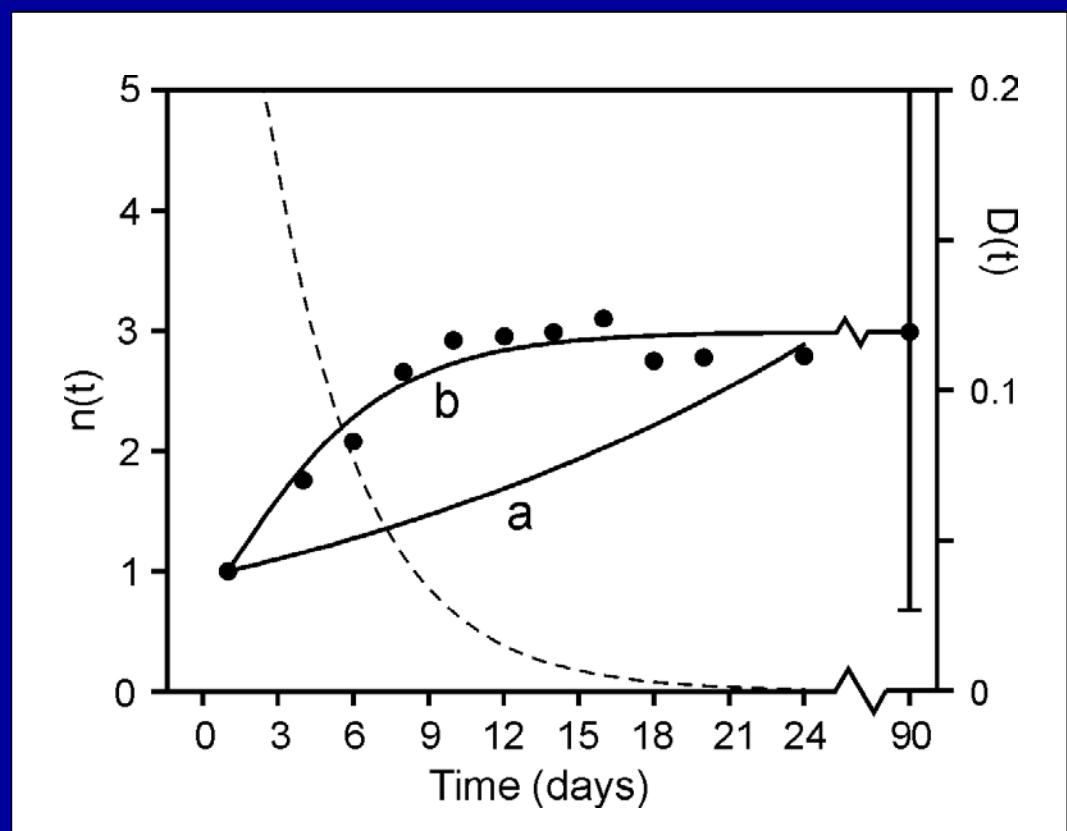
Growth function  $n(t)$  for

- (a) constant basal branching rate

$$D = 0.049$$

- (b) exponential basal branching rate

$$D(t) = 0.3e^{-(t-1)/3.7}$$



# Segment length distributions

*“Segment lengths determined by both neurite branching and elongation”*

Model approach:

- step 1: optimize branching process
- step 2: include elongation into model

Model optimized on S1-  
rat layer 2/3 pyramidal  
basal dendrites

## Branching:

$$B = 3.85$$

$$E = 0.74$$

$$S = 0.87$$

## Elongation:

$$V_{be} = 0.22 \mu\text{m/h}$$

$$V_e = 0.51 \mu\text{m/h}$$

$$CV = 0.28$$

## Empirical:

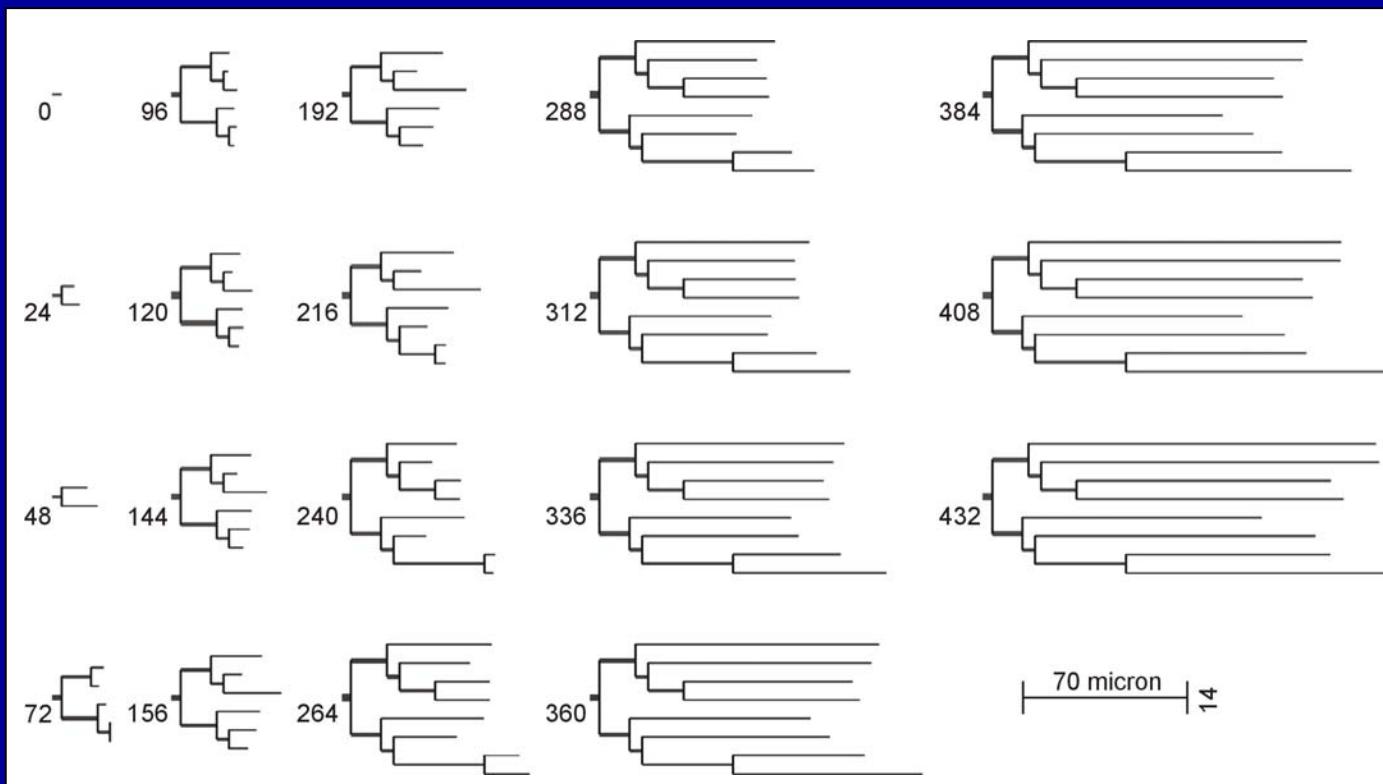
$$T_0 = -24 \text{ h}$$

$$T_{be} = 240 \text{ h}$$

$$T_e = 432 \text{ h}$$

This example  
for constant D

Growth of a model tree



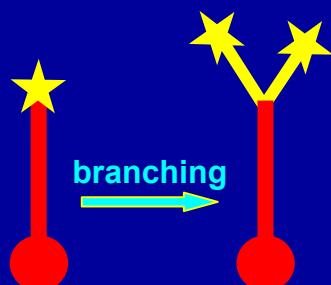
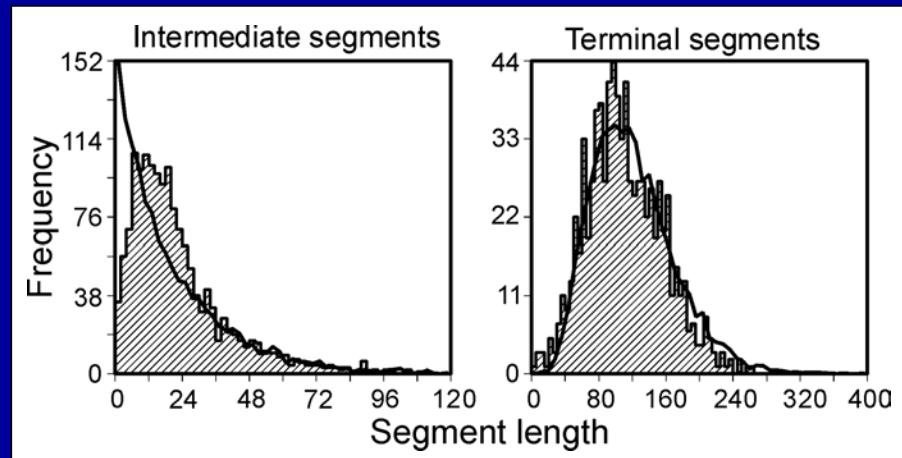
## Initial length after branching event

After branching event:

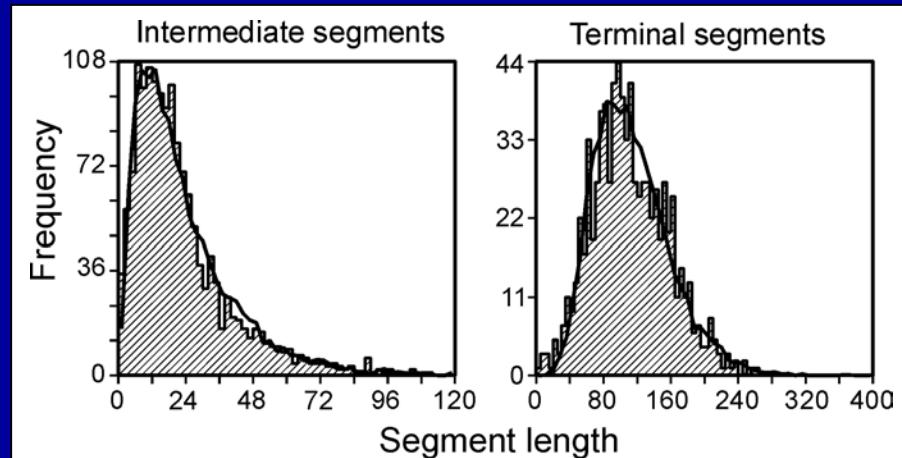


Daughter segments start with *zero* length

S1-rat layer 2/3 Pyramidal basal dendrites



Daughter segments start with *initial* length



# S1-rat layer 2/3 Pyramidal basal dendrites

## Branching:

$$B = 2.52$$

$$E = 0.73$$

$$S = 0.5$$

## Elongation:

$$l_{in} = 6 \mu\text{m}$$

$$\sigma(l_{in}) = 5$$

$$v_{be} = 0.22 \mu\text{m/h}$$

$$v_e = 0.51 \mu\text{m/h}$$

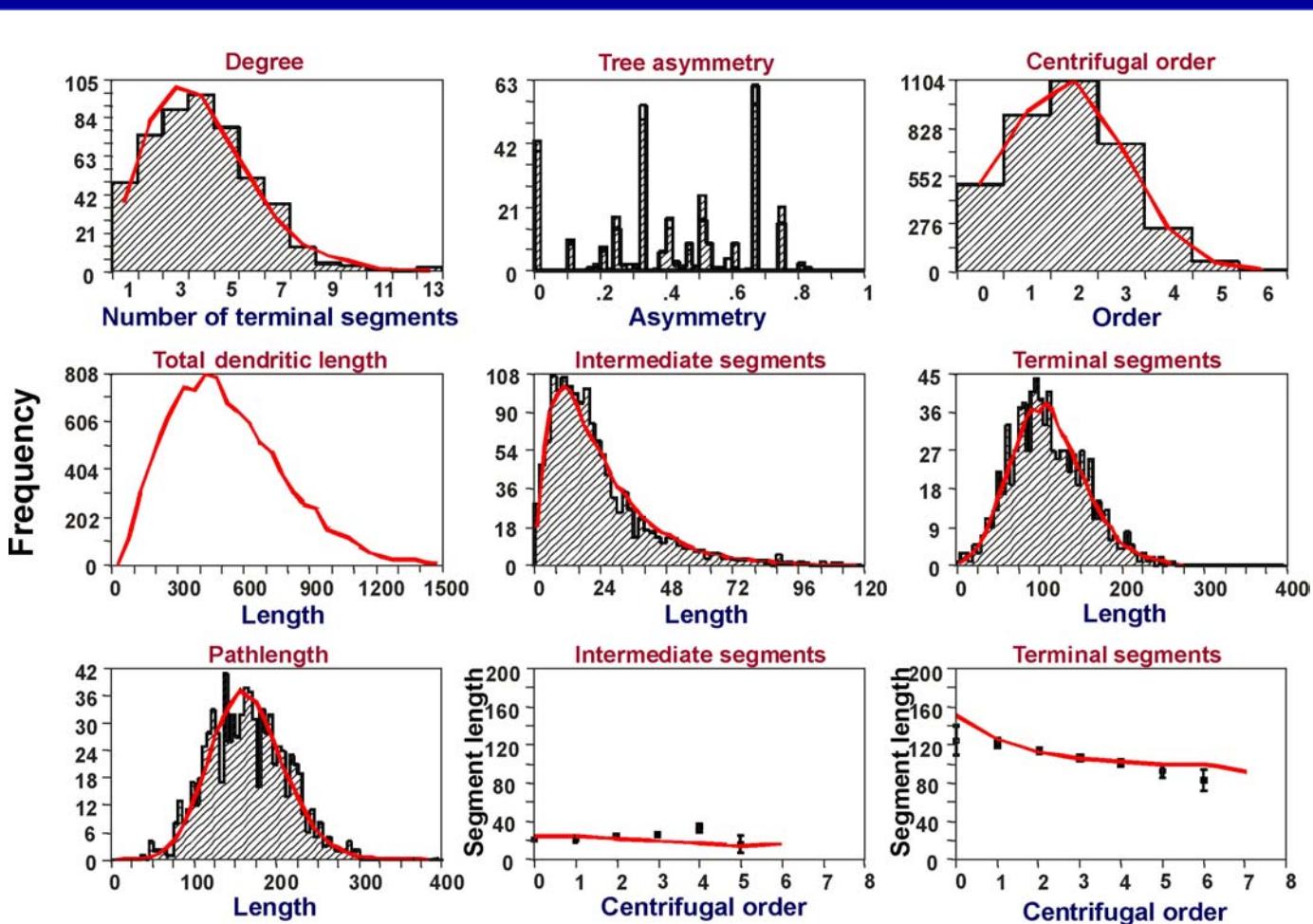
$$cv = 0.28$$

## Empirical:

$$T_0 = 24 \text{ h}$$

$$T_{be} = 240 \text{ h}$$

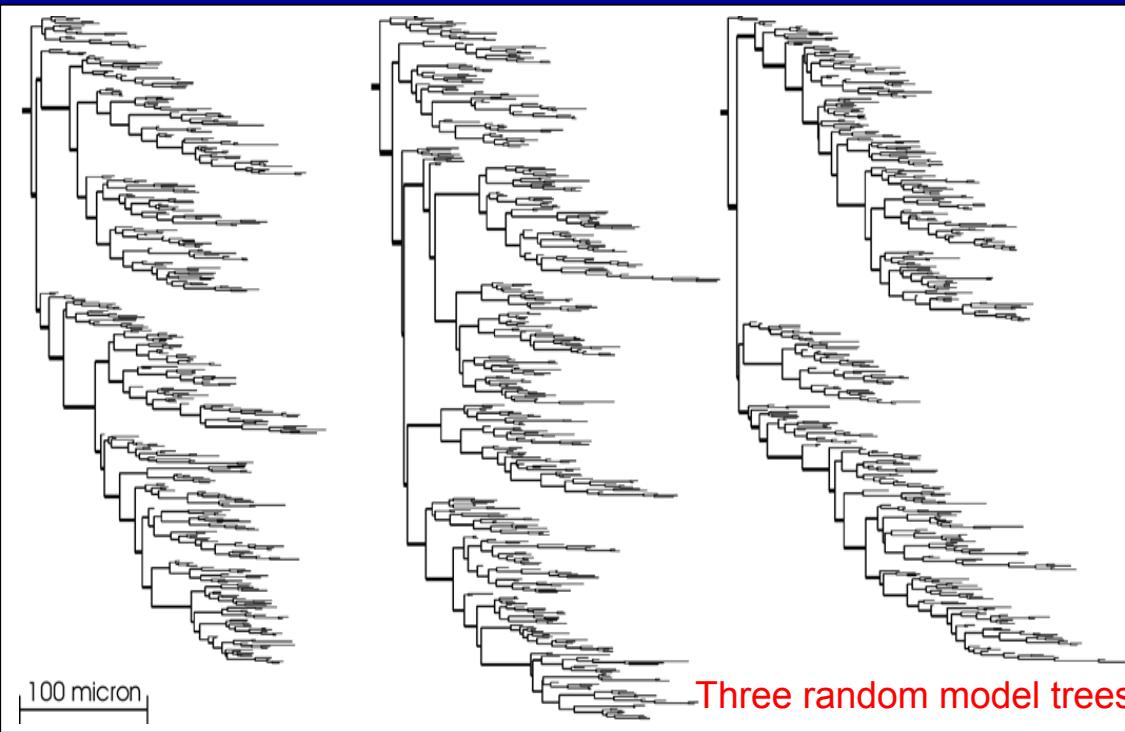
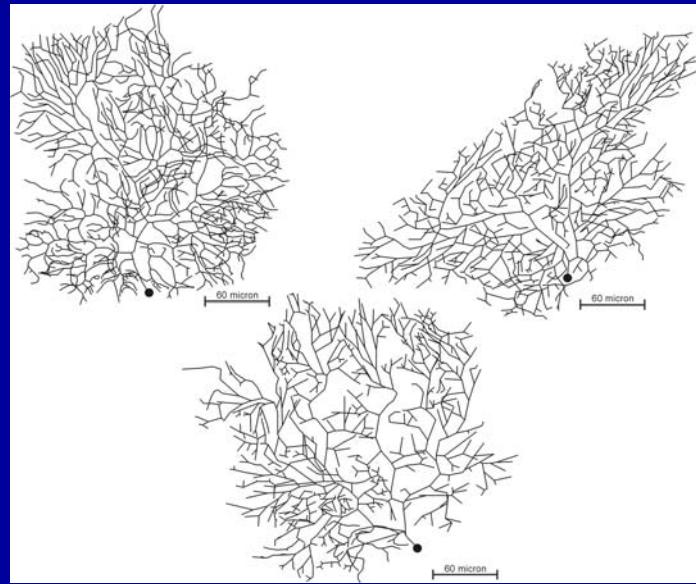
$$T_e = 432 \text{ h}$$



# Modeling guinea pig cerebellar Purkinje cell dendritic trees

## Reconstructions:

M. Rapp et al., J. Physiol. 474 (1994) 101-118



Three random model trees

## Branching:

$$B = 95$$

$$E = 0.69$$

$$S = -0.14$$

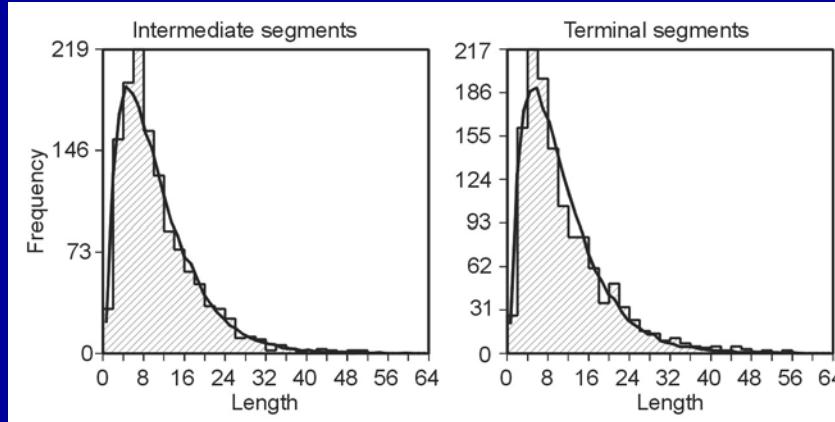
## Elongation:

$$\alpha(l_{in}) = 0.7 \mu\text{m}$$

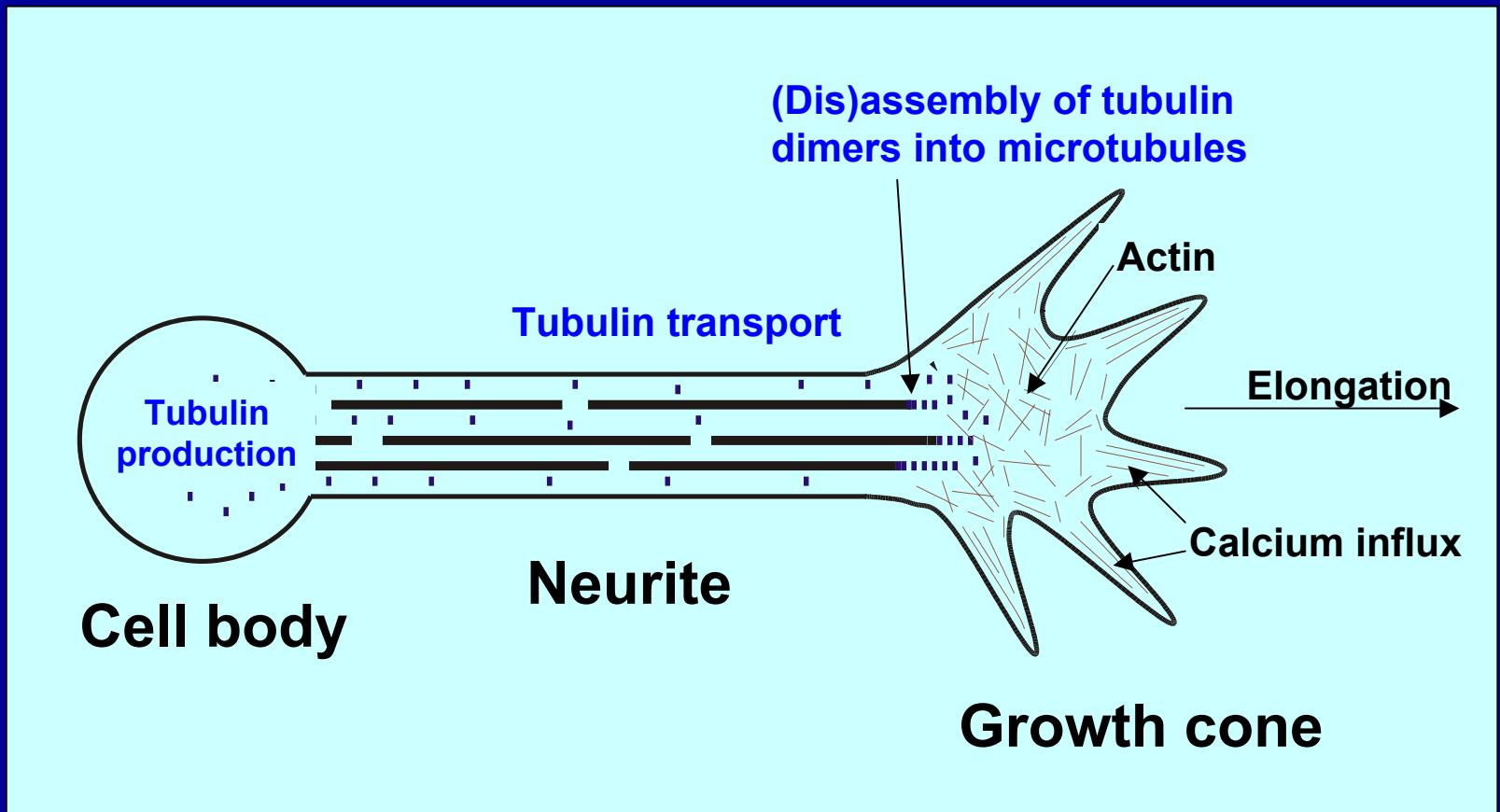
$$l_{in} = 10.63 \mu\text{m}$$

$$\sigma(l_{in}) = 7.53$$

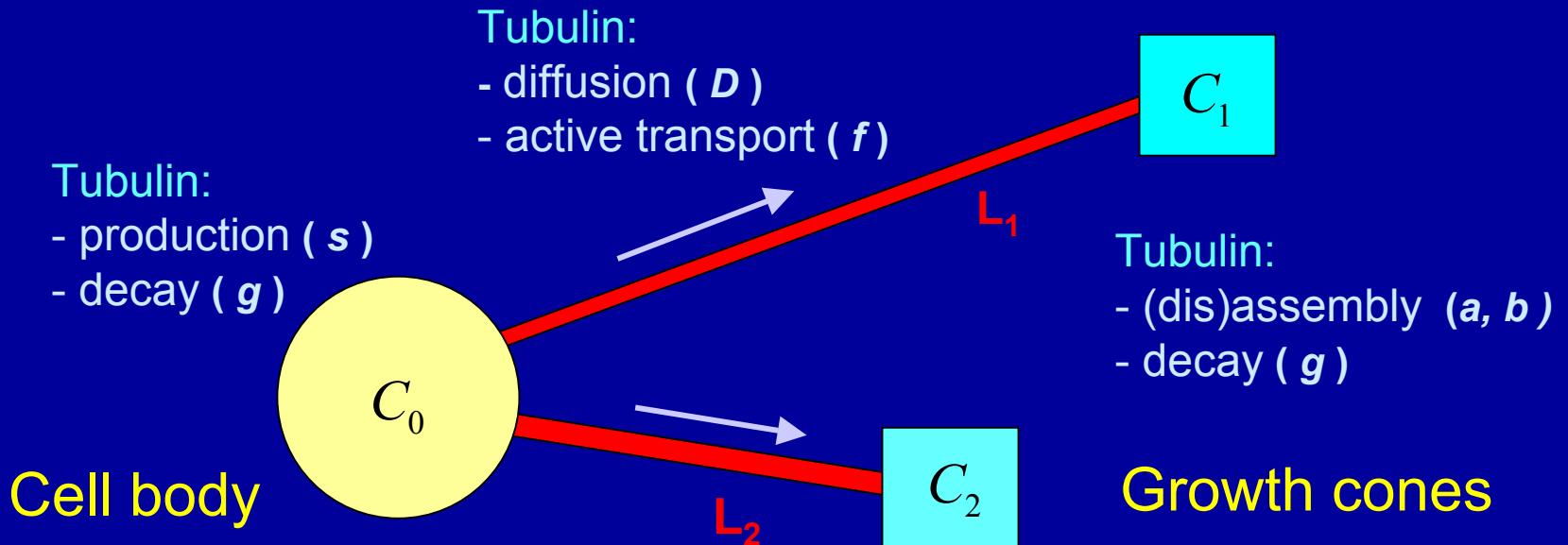
	Observed trees		Model trees	
Shape parameter	Mean	Sd	Mean	Sd
Degree	436	31.8	436	32
Asymmetry index	0.5		0.49	
Centrifugal order	13.7	5.1	13.8	5.9
Total length	9577	1105	9265	683
Terminal length	11.3	8.8	10.6	7.5
Intermediate length	10.6	7.5	10.6	7.6
Path length	189.3	64.1	166	66



# Elongation by polymerization of tubulin

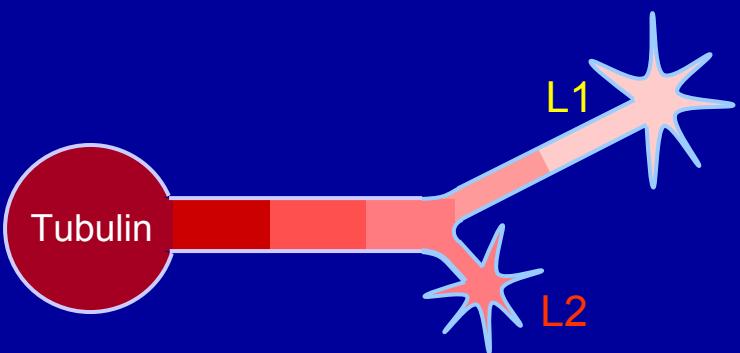


# Tubulin model

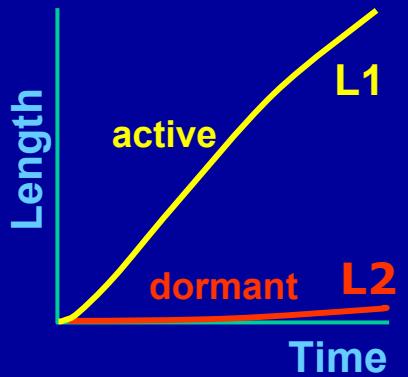


$$\frac{dL_i}{dt} = a_i C_i - b_i$$
$$\frac{dC_i}{dt} = b_i - a_i C_i + \frac{D}{L_i + k} (C_0 - C_i) + f C_0 - g C_i$$
$$\frac{dC_0}{dt} = s - \sum_{i=1}^n \frac{D}{L_i + k} (C_0 - C_i) - \sum_{i=1}^n f C_0 - g C_0$$

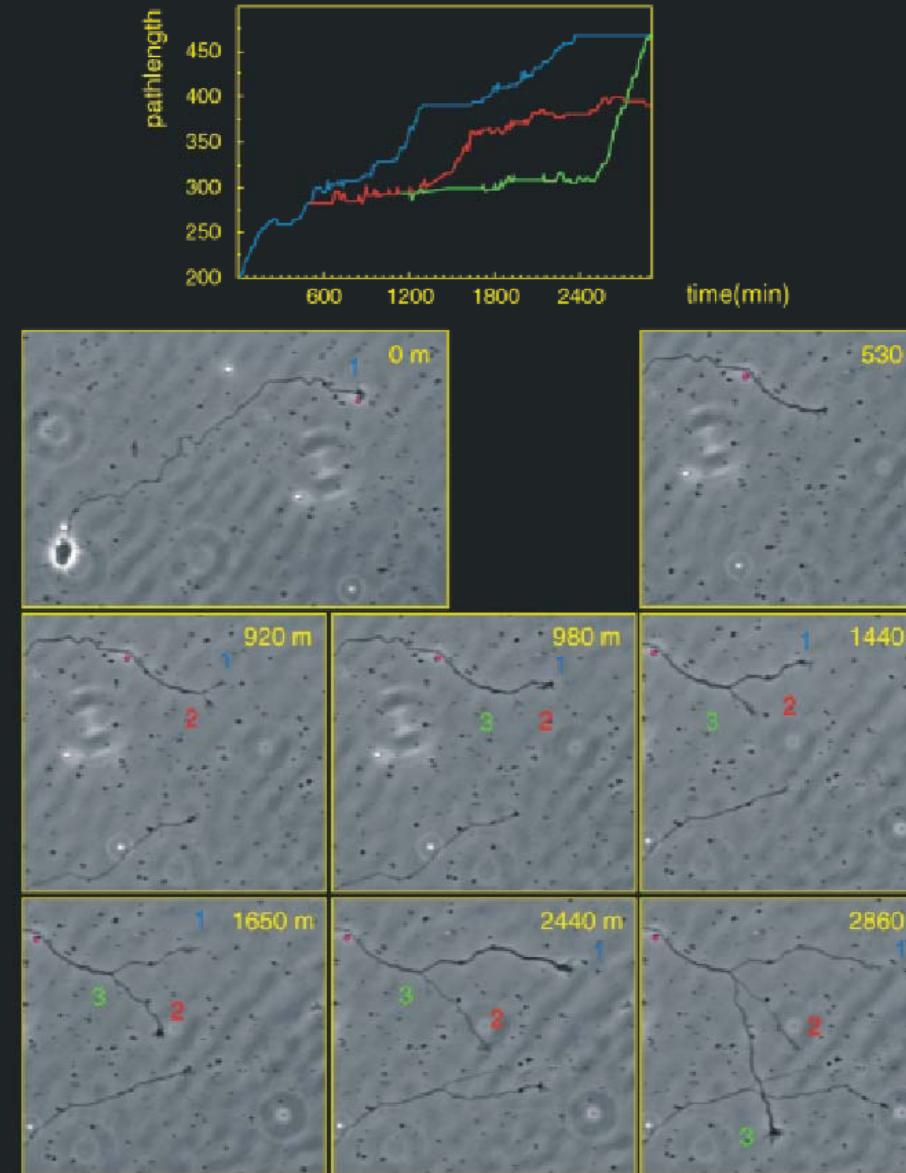
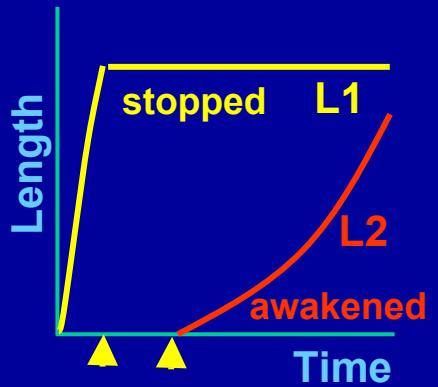
# Competition for tubulin



Difference in assembly / disassembly rates results in competition



Stop of L1 outgrowth activates “dormant” growth cone after characteristic delay



# Development of synaptic connectivity

• Synapses form between neurons

• Neurons receive information from other neurons via synapses

• Neurons send information to other neurons via synapses

• Synapses are formed by the growth of neurites

• Synapses are formed by the growth of neurites

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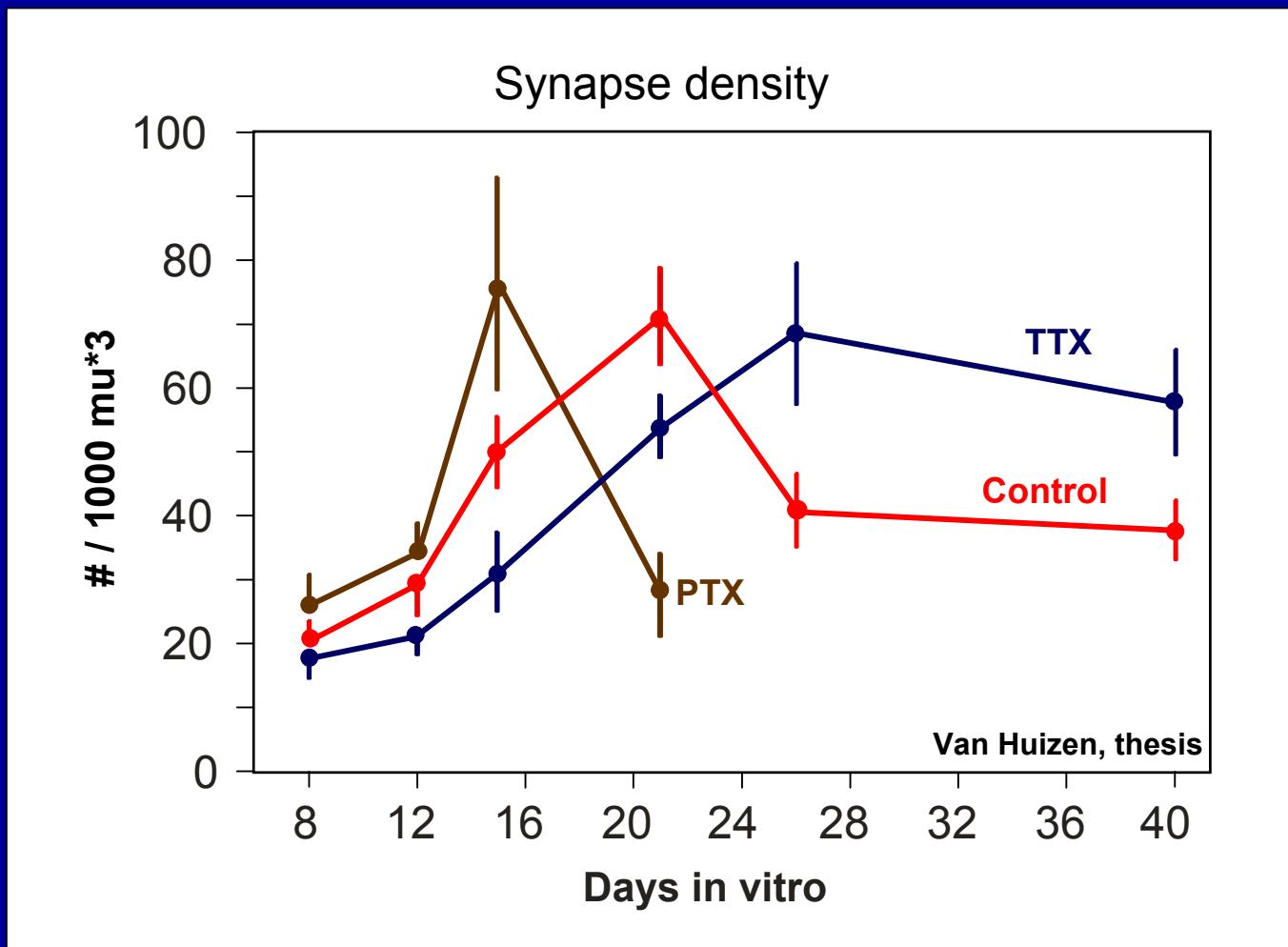
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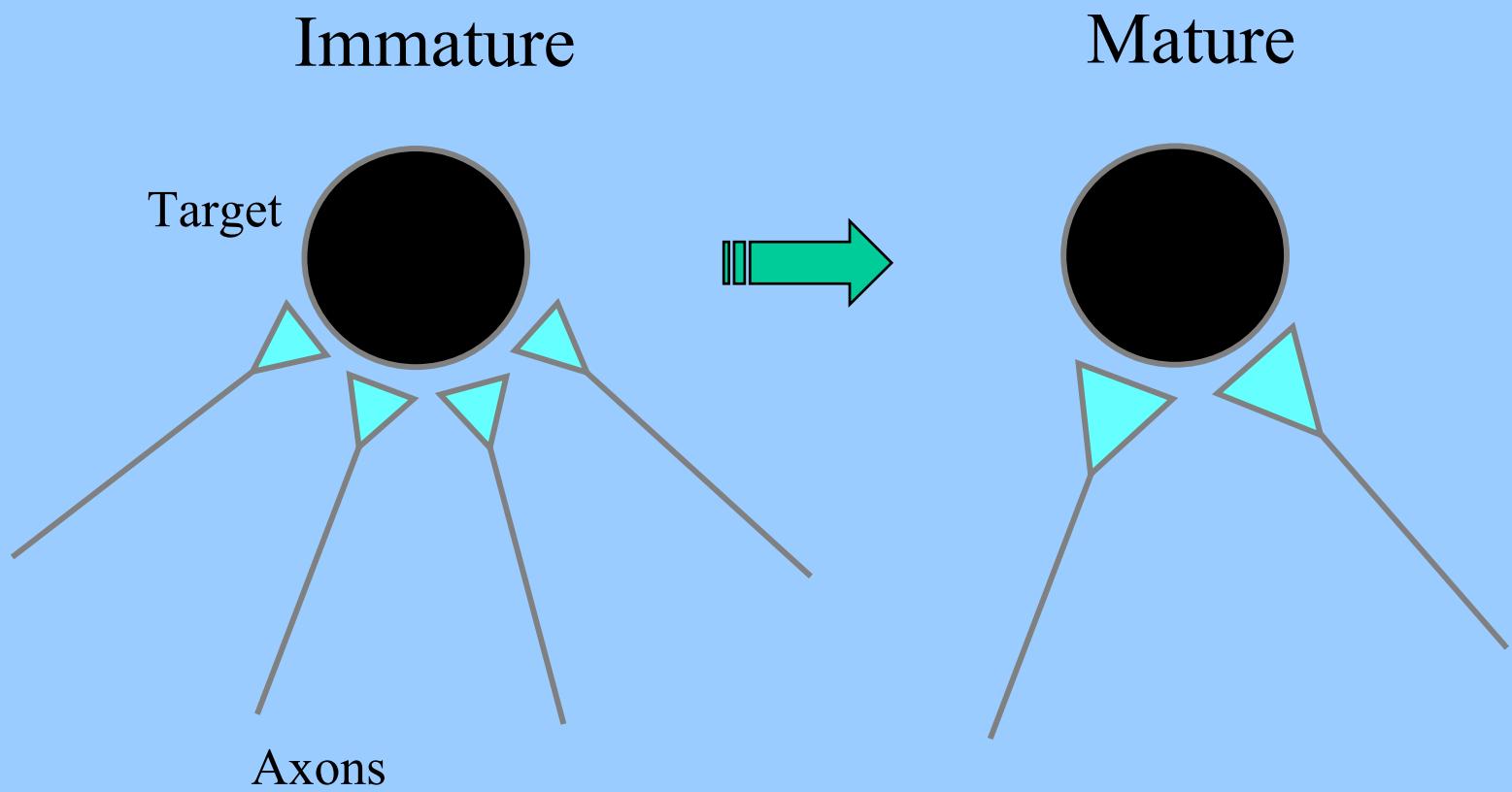
• Synapses are formed by the growth of neurites

• Synapses are formed by the growth of neurites

# Developing neuronal networks show an overshoot in number of synaptic connections

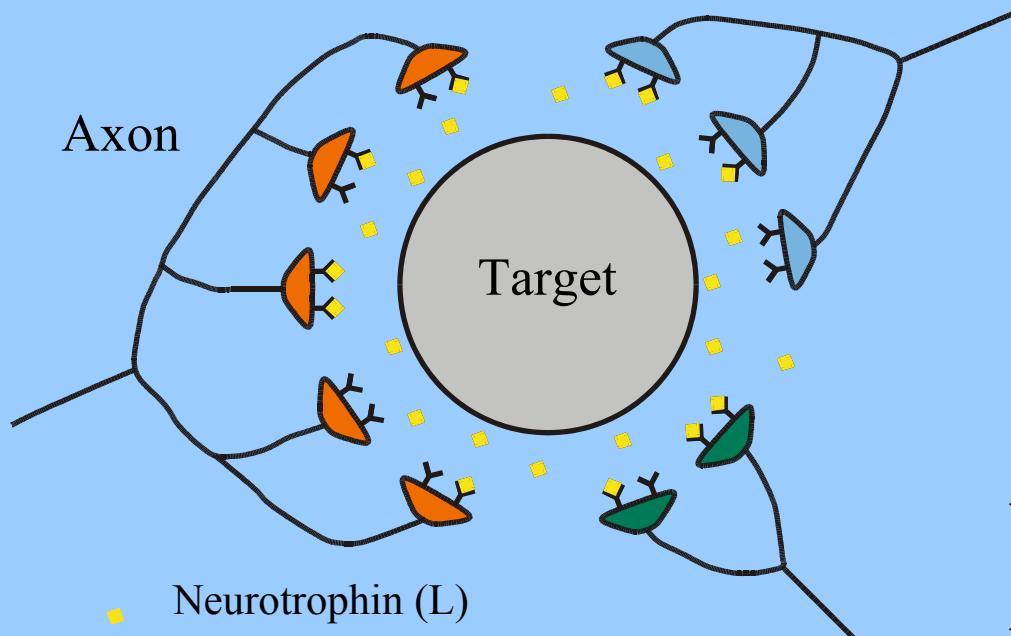


# Elimination of connections

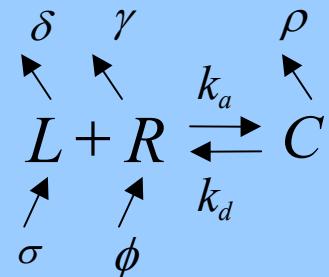


# Van Ooyen & Willshaw's model

*“competition for target-derived neurotrophins”*



- Neurotrophin (L)
- ▲ Receptor (R)
- ◆ Neurotrophin-receptor complex (C)



$$\begin{aligned}\frac{dC_i}{dt} &= (k_{a,i} LR_i - k_{d,i} C_i) - \rho_i C_i \\ \frac{dR_i}{dt} &= \varphi_i - \gamma_i R_i - (k_{a,i} LR_i - k_{d,i} C_i) \\ \frac{dL}{dt} &= \sigma - \delta L - \sum_{i=1}^n (k_{a,i} LR_i - k_{d,i} C_i) / v\end{aligned}$$

# Effects of neurotrophins

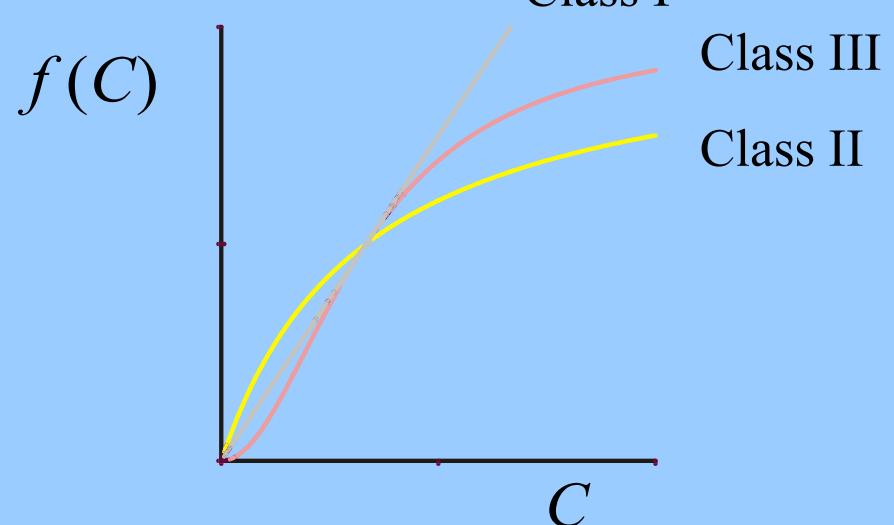
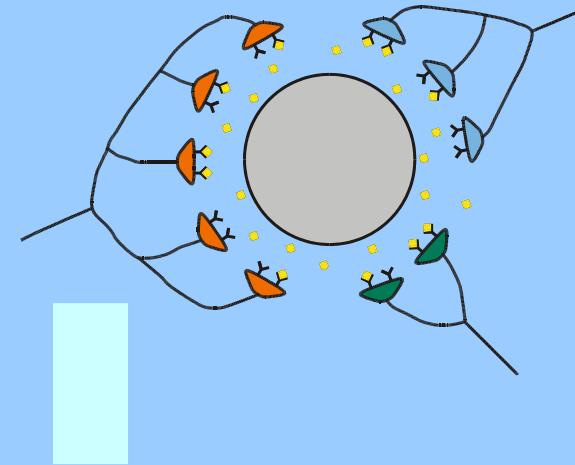
*Bound neurotrophins enhance axonal branching*

➡ more receptors

$$\text{Thus, } \varphi_i = f_i(C_i)$$

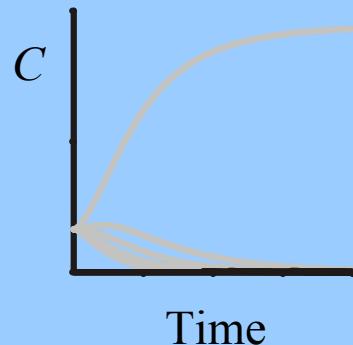
But lags behind:

$$\tau \frac{d\varphi_i}{dt} = f_i(C_i) - \varphi_i$$

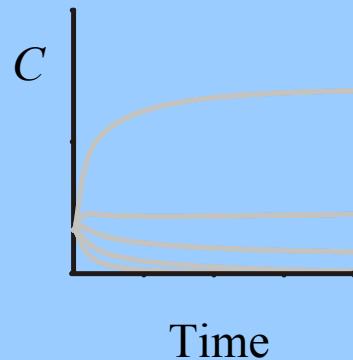


# Innervation patterns

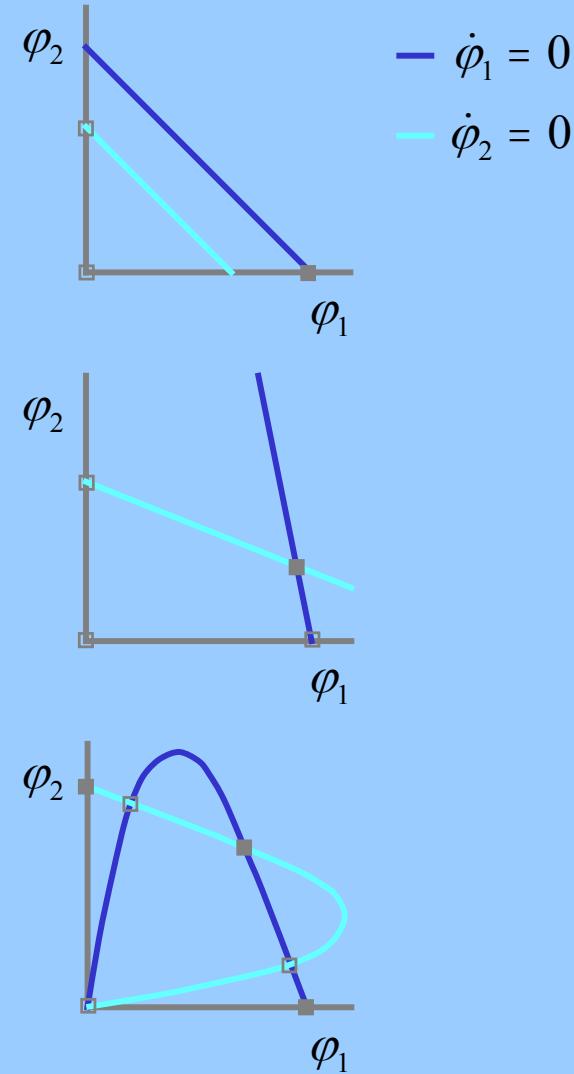
- **Class I:** Single



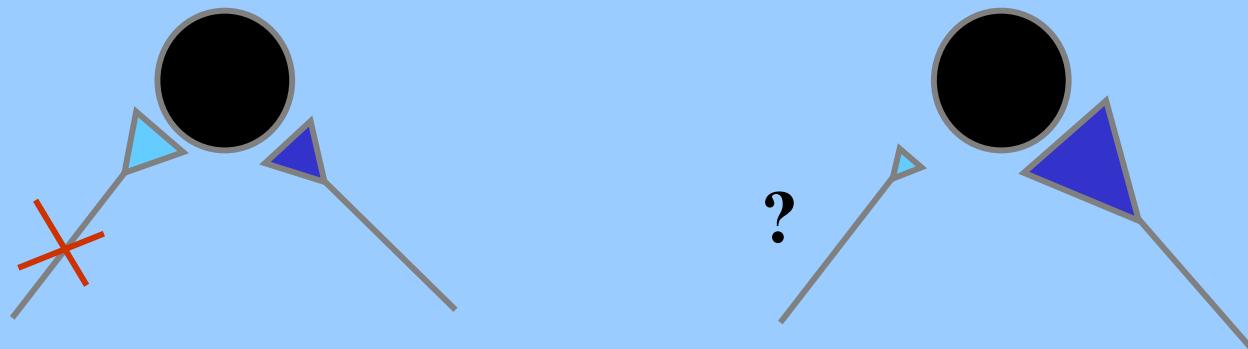
- **Class II:** Multiple,  
Single



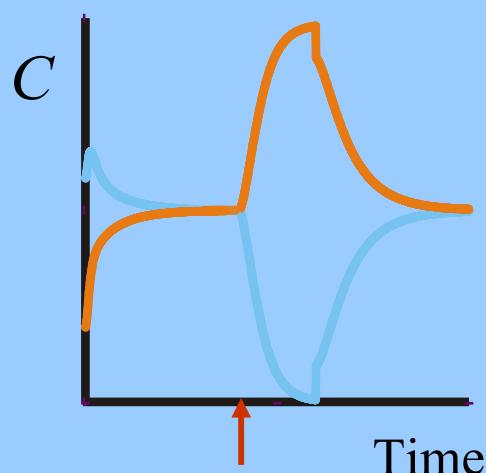
- **Class III:** Multiple and single,  
Single



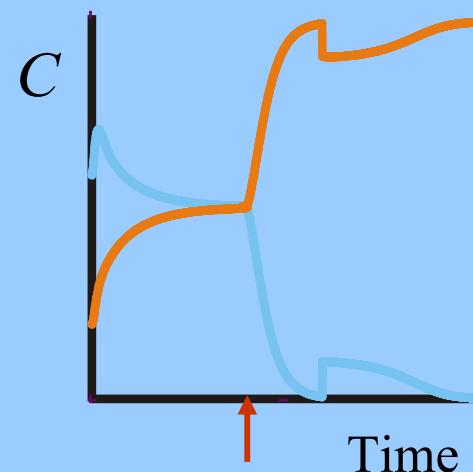
# Regeneration



Class II

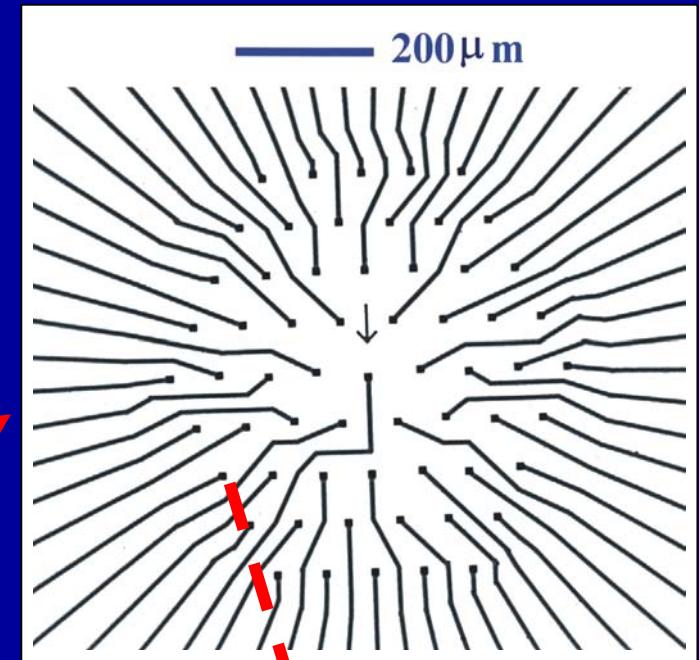
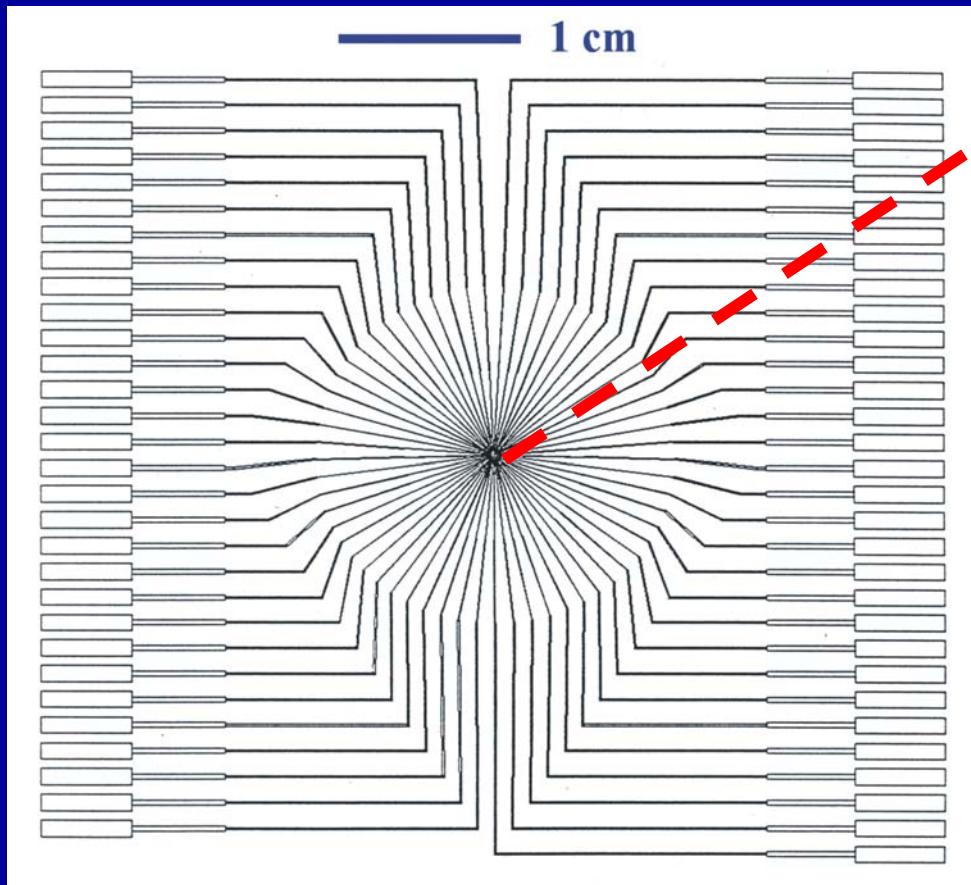


Class III



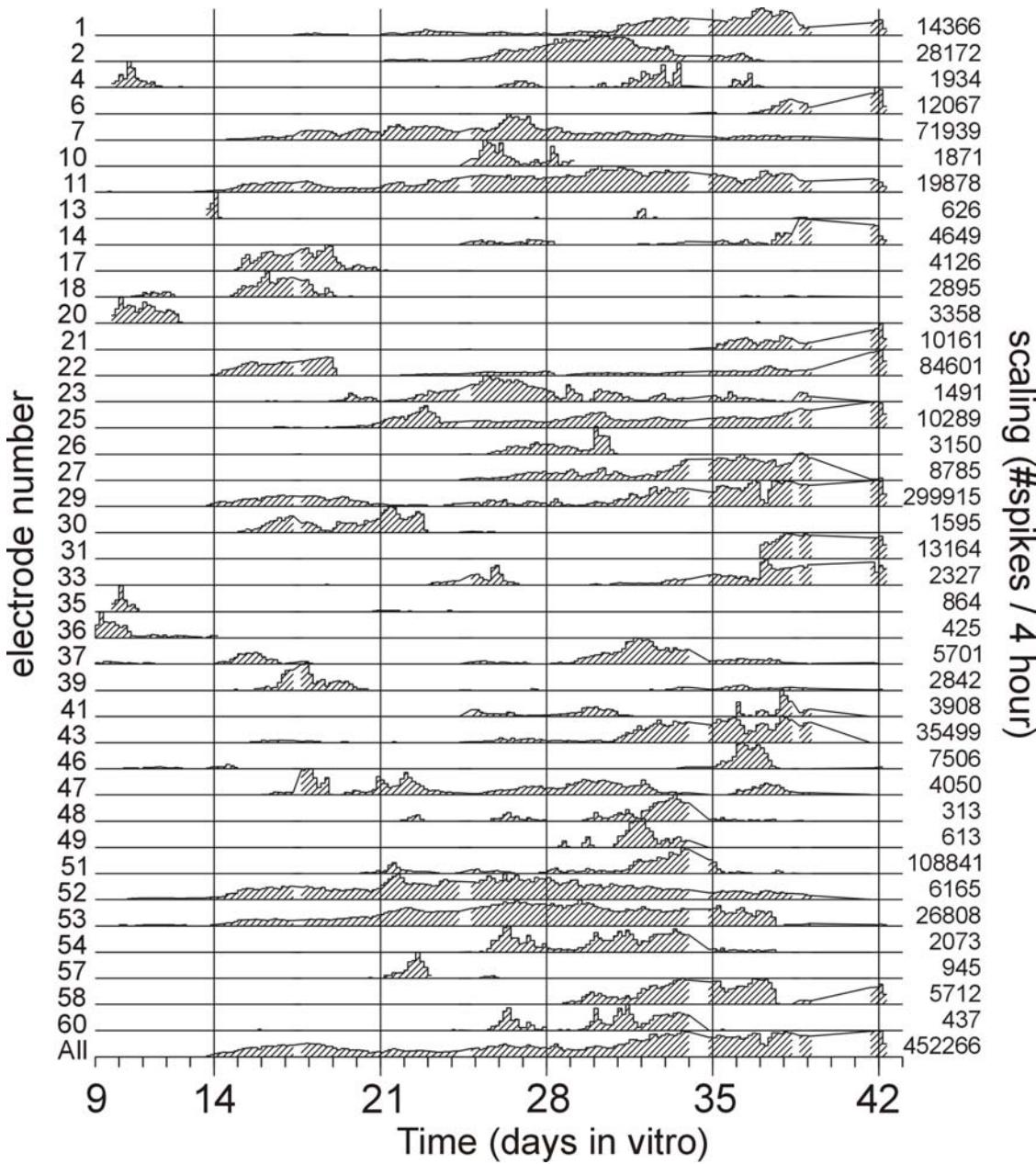
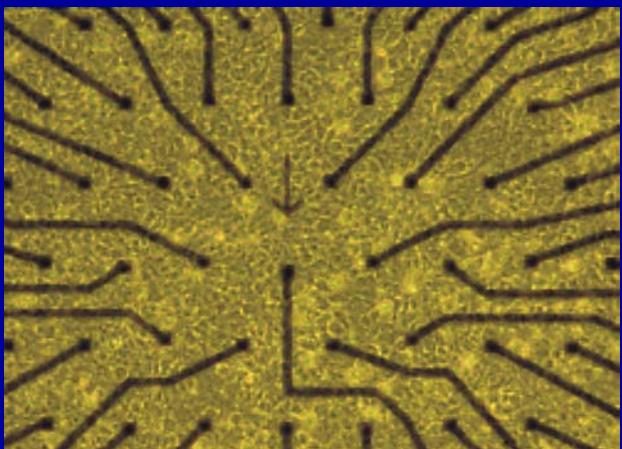
# Spontaneous bioelectric activity in developing neuronal networks in vitro

# Multi-electrode array



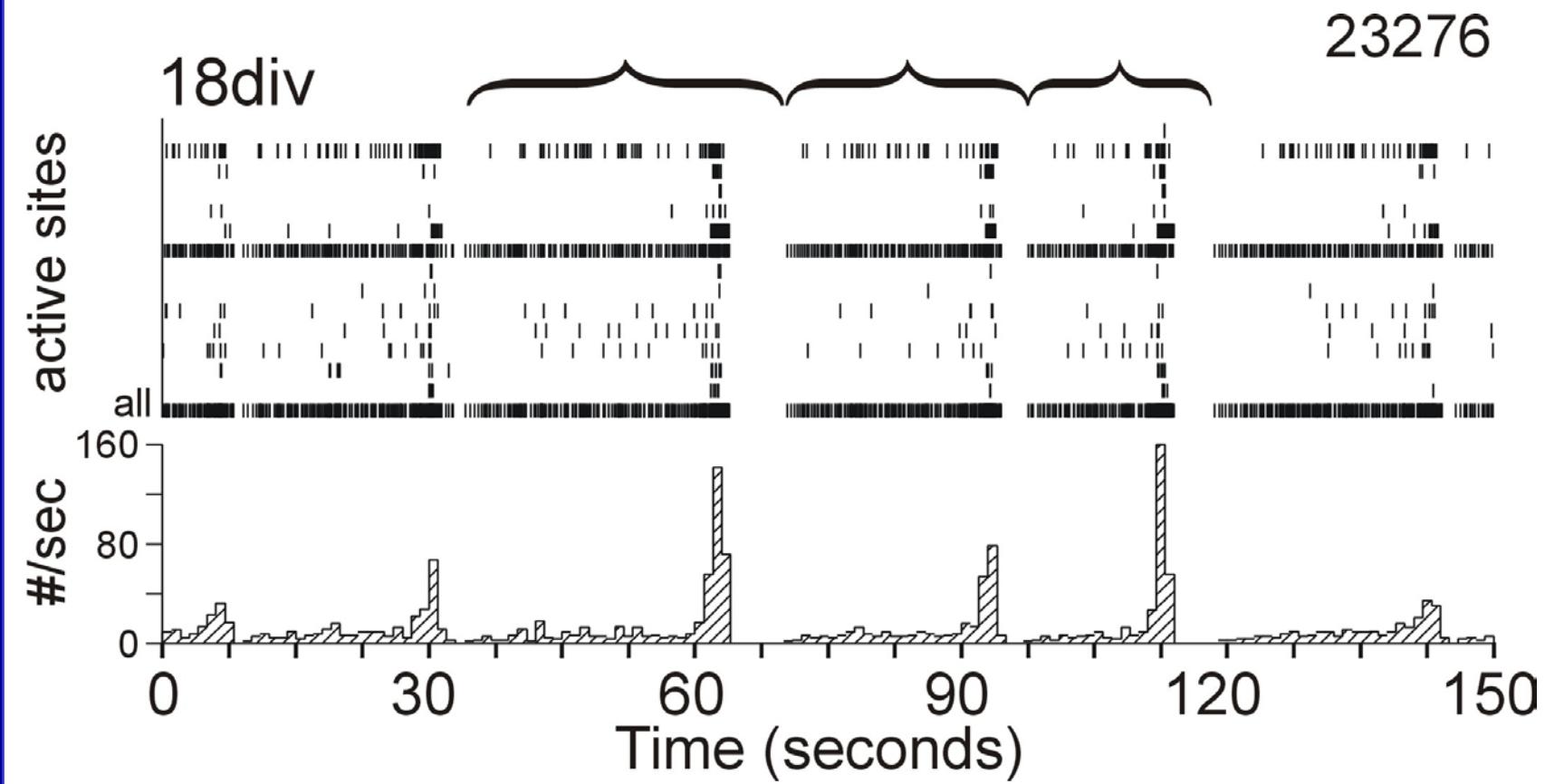
# Long-term longitudinal recording of firing activity

Dissociated rat cortical  
culture on a  
multi-electrode array



18 DIV

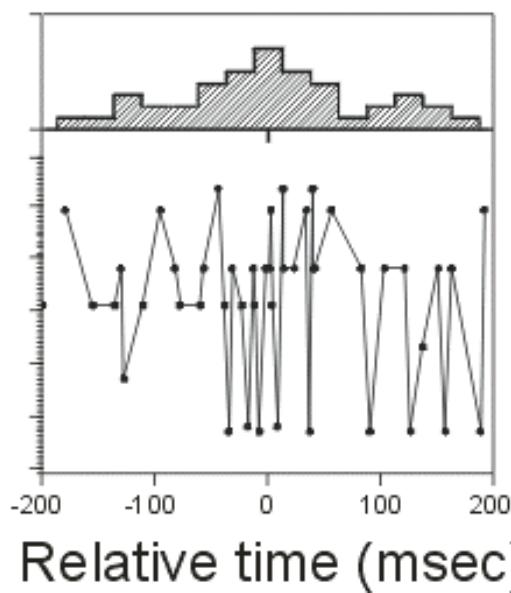
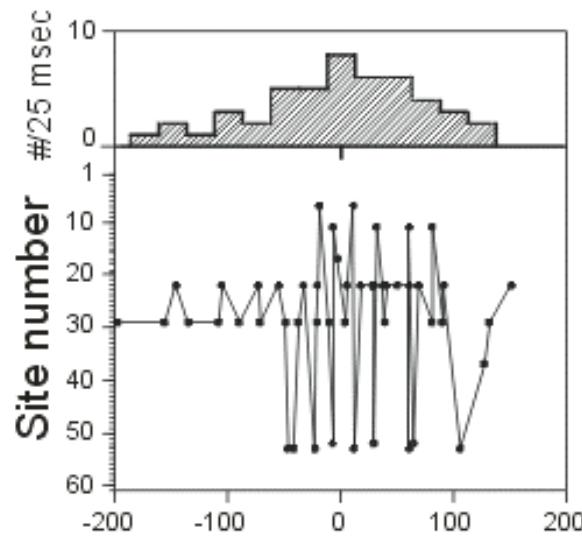
## Active / quiescent phases and network bursts



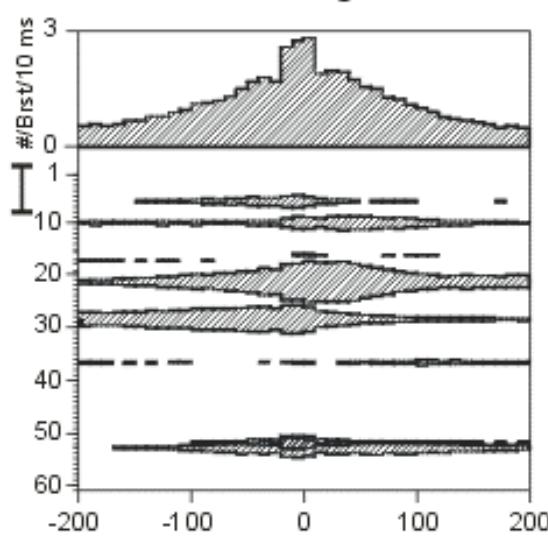
## Probabilistic structure of network bursts

15 DIV

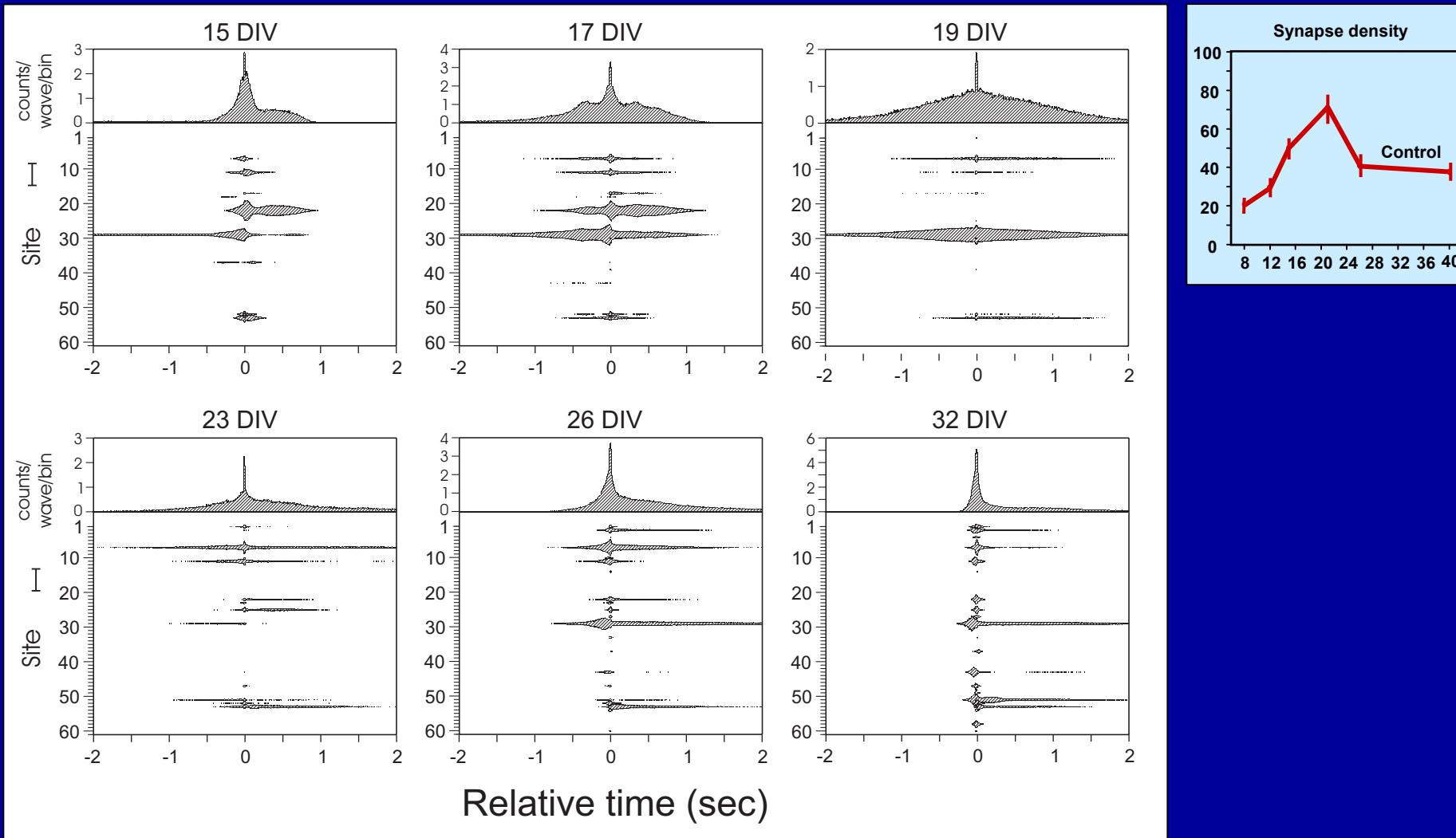
Individual network bursts



Summation of aligned bursts



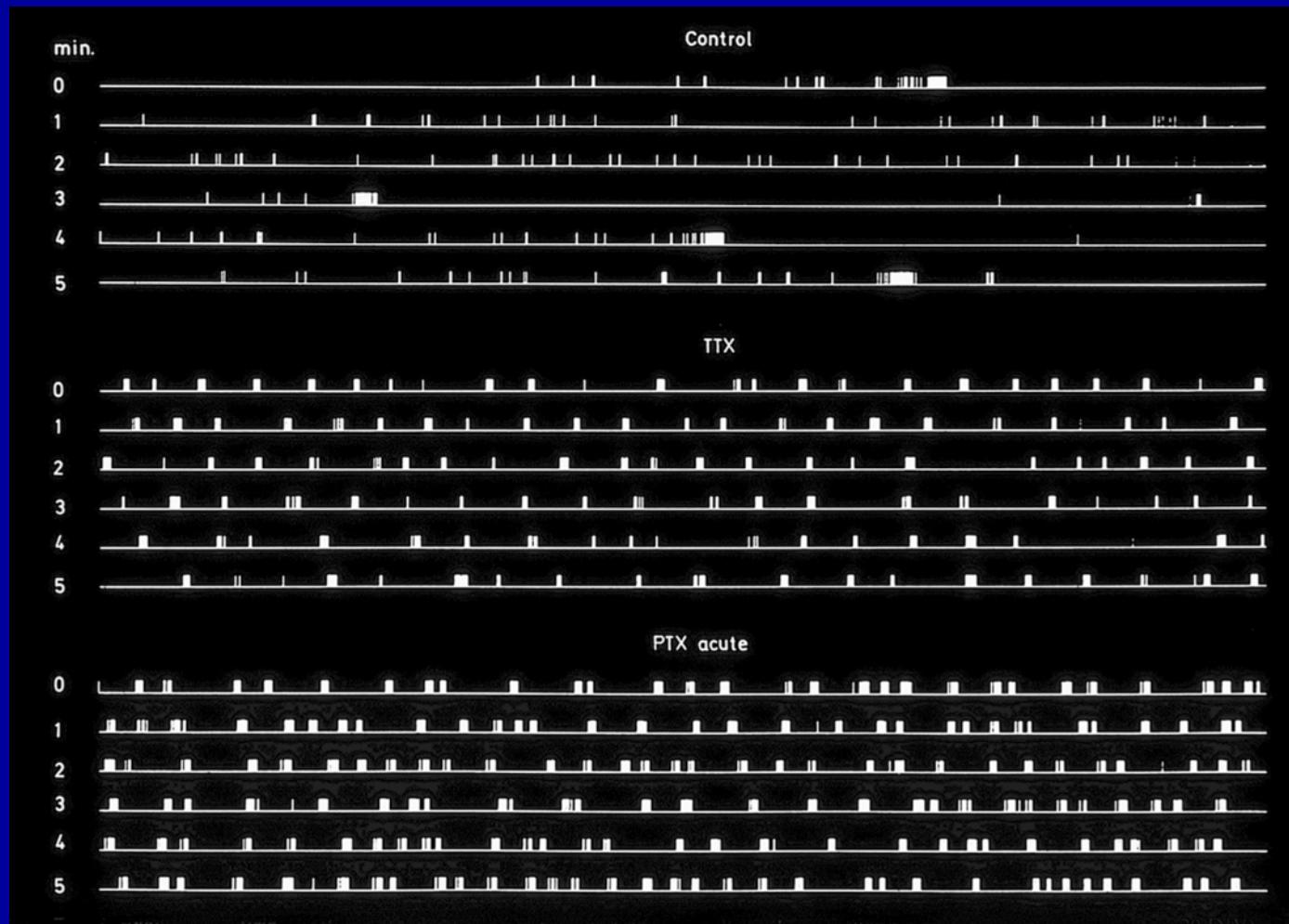
# Developmental changes in network bursts



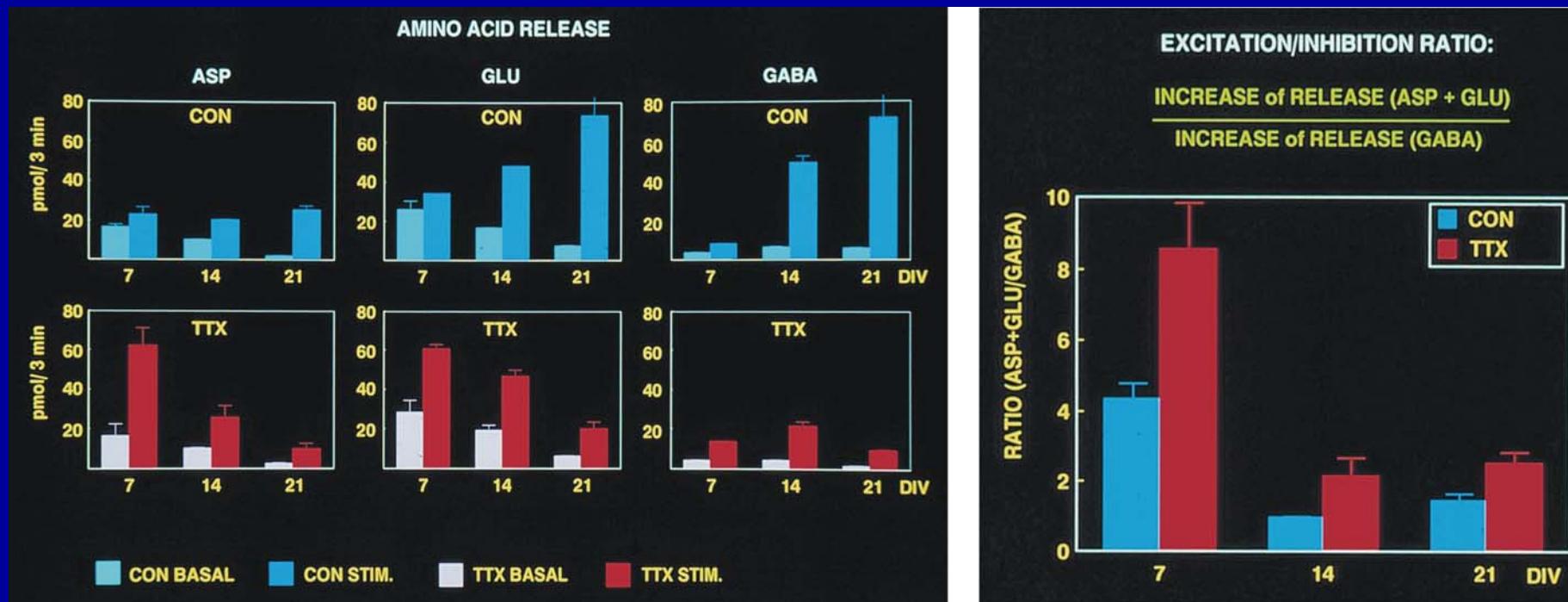
# Balance of excitation and inhibition

# Chronic silencing of spontaneous activity induces disinhibition

Control condition  
After chronic TTX application  
During acute PTX application



# Chronic suppression of spontaneous activity increases the excitation / inhibition ratio



Working hypothesis:  
activity-dependent regulation of excitatory and inhibitory  
network development

